Math 1 Practice Problems I Solutions

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Answers

This page contains answers only. Detailed solutions are on the following pages. All plots were made using `http://www.desmos.com/`.

1. $h(1) = 1, h(-1) = -2, h(2) = \frac{5}{2}, h(\frac{1}{2}) = \frac{5}{2}, h(x) = x + \frac{1}{x}, h(\frac{1}{x}) = x + \frac{1}{x}$
2. $f(-5) = -15, f(0) = 1, f(1) = 2, f(2) = 3, f(5) = 9$
3. See detailed solutions for plots.
   (a) Domain: $(-\infty, \infty)$
   Range: $\{1\} \cup (2, \infty)$
   (b) Domain: $(-\infty, \infty)$
   Range: $(-\infty, 4]$
   (c) Domain: $(-\infty, \infty)$
   Range: $[0, \infty)$
4. (a) $\mathbb{R}$ or $(-\infty, \infty)$
   (b) $[-3, 3]$
   (c) $(-\infty, -\pi) \cup (-\pi, -1) \cup (-1, 0) \cup (0, \infty)$
   (d) $(-1, \infty)$
   (e) $(-\infty, -4) \cup (-4, \infty)$
   (f) $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)$
   (g) $(-\infty, -1] \cup [1, 4]$
5. (a) $-\frac{1}{15}$
   (b) $-\frac{1}{2(2+h)}$
6. $f(f(x)) = x, f(g(x)) = \frac{1}{2x+4}, g(f(x)) = \frac{2}{x} + 4, g(g(x)) = 4x + 12$
7. $f \circ g \circ h(x) = 1 + \sqrt{x}$
8. (a) Horizontal stretch (divide x values by $\frac{1}{4}$)
   (b) Vertical reflection, horizontal compression (divide x values by 2)
   (c) Shift right 4, up $\frac{3}{4}$
9. See detailed solutions for plots.
10. (a) Odd
    (b) Even
    (c) Neither
    (d) Neither
11. Show $f(g(x)) = g(f(x)) = x$
12. (a) $f^{-1}(x) = \frac{1}{2}x^2 + \frac{1}{2}$
    (b) $g^{-1}(x) = \frac{1}{x} - 2$

Detailed Solutions

1. Let $h(t) = t + \dfrac{1}{t}$. Find $h(1), h(-1), h(2), h(\frac{1}{2}), h(x), h(\frac{1}{x})$.

   Solution.

$$\begin{aligned} h(1) &= 1 + \frac{1}{1} = 2 \\ h(-1) &= -1 + \frac{1}{-1} = -2 \\ h(2) &= 2 + \frac{1}{2} = \frac{5}{2} \\ h\left(\frac{1}{2}\right) &= \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = \frac{5}{2} \\ h(x) &= x + \frac{1}{x} \\ h\left(\frac{1}{x}\right) &= \frac{1}{x} + \frac{1}{\frac{1}{x}} = \frac{1}{x} + x \end{aligned}$$

   □

2. Evaluate $f(-5), f(0), f(1), f(2), f(5)$ where $f(x) = \begin{cases} 3x, & x < 0 \\ x + 1, & 0 \le x \le 2 \\ (x-2)^2, & x > 2 \end{cases}$.

   Solution.

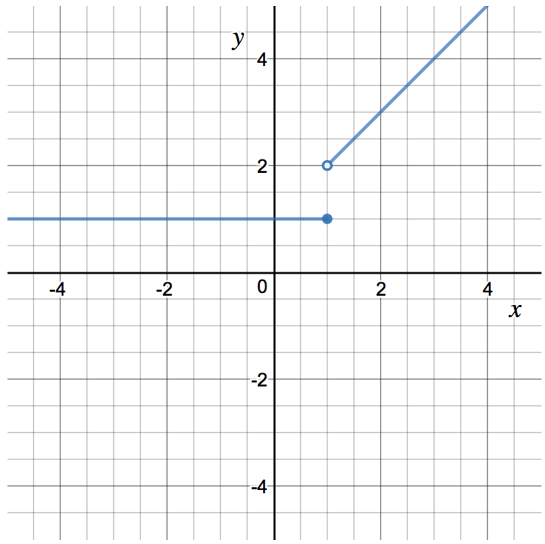
$$\begin{aligned} f(-5) &= 3(-5) = -15 \\ f(0) &= 0 + 1 = 1 \\ f(1) &= 1 + 1 = 2 \\ f(2) &= 2 + 1 = 3 \\ f(5) &= (5-2)^2 = 3^2 = 9 \end{aligned}$$

   □

3. Sketch the graph and find the domain and range for each of the following functions:

   (a) $f(x) = \begin{cases} 1, & x \le 1 \\ x + 1, & x > 1 \end{cases}$

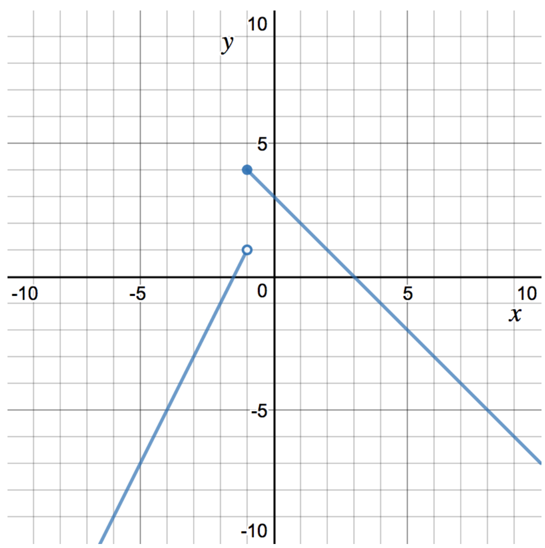
   Solution. See the plot below. The domain is $(-\infty, \infty)$ and the range is $\{1\} \cup (2, \infty)$.



□

(b) $f(x) = \begin{cases} 2x+3, & x < -1 \\ 3-x, & x \geq -1 \end{cases}$

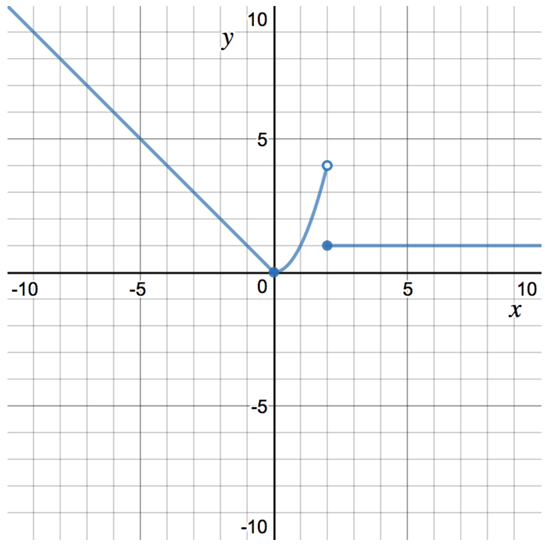
Solution. See the plot below. The domain is $(-\infty, \infty)$ and the range is $(-\infty, 4]$.



□

(c) $g(t) = \begin{cases} -t, & t < 0 \\ t^2, & 0 \leq t < 2 \\ 1, & t \geq 2 \end{cases}$

Solution. See the plot below. The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.



□

4. Find the domain of the following functions:

   (a) $f(x) = \sqrt[3]{x^{10} - 11}$

   Solution. We don't have any issues with odd roots, so the domain here is $\mathbb{R}$ or $(-\infty, \infty)$. □

   (b) $f(x) = \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x+\pi}$

   Solution. Find the domain of each piece separately, then combine at the end.
   For $\frac{1}{x}$, we cannot have the denominator equal to 0, hence $x \neq 0$.
   For $\frac{1}{x+1}$, we cannot have the denominator equal to 0, hence $x + 1 \neq 0$, or $x \neq -1$.
   For $\frac{1}{x+\pi}$, we cannot have the denominator equal to 0, hence $x + \pi \neq 0$, or $x \neq -\pi$.
   Therefore the domain is $(-\infty, -\pi) \cup (-\pi, -1) \cup (-1, 0) \cup (0, \infty)$. □

   (c) $h(t) = \sqrt[4]{9 - t^2}$

   Solution. We can only take the even root of a nonnegative number, hence $9 - t^2 \geq 0$. Factor:

$$(3 - t)(3 + t) \geq 0 \tag{1}$$

   Ignoring the inequality and setting equal to 0 yields $t = 3, t = -3$. Therefore we test left of $t = -3$, between $t = -3$ and $t = 3$, and to the right of $t = 3$.
   Left of $t = -3$: We substitute a number left of -3, e.g. $t = -4$, into (1) which yields

$$(-3 - (-4))(-3 + (-4)) \geq 0,$$

which is a false statement since a negative number is not greater than or equal to 0. Therefore we cannot have any numbers left of -3.

Between $t = -3$ and $t = 3$: We substitute a number between -3 and 3, e.g. $t = 0$, into (1) which yields

$$(3-0)(3+0) \geq 0$$

which is a true statement. Therefore we can have numbers between -3 and 3.

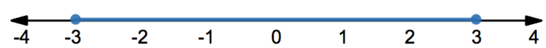
Right of $t = 3$: We substitute a number right of 3, e.g. $t = 4$, into (1) which yields

$$(3-4)(3+4) \geq 0$$

which is a false statement. Therefore we cannot have any numbers of left of 3.

Thus the domain is $[-3, 3]$

For inequalities involving polynomials, it is often helpful to test and plot the domain using a number line as seen below:



□

(d) $g(x) = \dfrac{x}{\sqrt{x+1}}$

Solution. We cannot have 0 on the denominator and the number inside of a square root must be nonnegative, therefore we must have

$$x + 1 > 0 \quad \Rightarrow \quad x > -1,$$

thus the domain is $(-1, \infty)$. □

(e) $f(x) = \dfrac{\sqrt[3]{2x+1}}{\sqrt[3]{2x}+2}$

Solution. We don't have any issues with the odd root on either the numerator or denominator. We cannot have 0 on the denominator, hence

$$\sqrt[3]{2x} + 2 \neq 0 \quad \Rightarrow \quad \sqrt[3]{2x} \neq -2 \quad \Rightarrow \quad 2x \neq -8 \quad \Rightarrow \quad x \neq -4,$$

thus the domain is $(-\infty, -4) \cup (-4, \infty)$. □

(f) $g(u) = \dfrac{2u^2 + 5u + 3}{2u^2 - 5u - 3}$

Solution. We cannot have 0 on the denominator:

$$2u^2 - 5u - 3 \neq 0 \quad \Rightarrow \quad (u-3)(2u+1) \neq 0 \quad \Rightarrow \quad u \neq 3 \text{ and } u \neq -\frac{1}{2}$$

thus the domain is $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)$. □

(g) $F(x) = \sqrt{4-x} + \sqrt{x^2-1}$

Solution. Find the domain of each piece separately, then combine at the end.
For $\sqrt{4-x}$, we need $4 - x \geq 0$, or $x \leq 4$.
For $\sqrt{x^2-1}$, we need $x^2 - 1 \geq 0$. Just as in part (b) above, we factor and test points on a number line:

$$(x-1)(x+1) \geq 0$$

Setting equal to 0 yields $x = -1, x = 1$. Test left of $x = -1$, between $x = -1$ and $x = 1$, and right of $x = 1$. Testing on the left of $x = -1$ and right of $x = 1$ yields true statements, therefore the domain is $x \leq -1$ and $x \geq 1$.
Combining the domain of both square roots together yields $(-\infty, -1] \cup [1, 4]$.

□

5. Find the average rate of change of the function $f(x) = \frac{1}{x}$ on the following intervals:

(a) $[3, 5]$

Solution.

$$\frac{f(5) - f(3)}{5-3} = \frac{\frac{1}{5} - \frac{1}{3}}{2} = \frac{\frac{3}{15} - \frac{5}{15}}{2} = \frac{-2}{15} \cdot \frac{1}{2} = -\frac{1}{15}$$

□

(b) $[2, 2+h]$

Solution.

$$\frac{f(2+h) - f(2)}{2+h-2} = \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{\frac{2-(2+h)}{2(2+h)}}{h} = -\frac{h}{2(2+h)} \cdot \frac{1}{h} = -\frac{1}{2(2+h)}$$

□

6. Find $f(f(x)), f(g(x)), g(f(x)), g(g(x))$ where $f(x) = \frac{1}{x}$, $g(x) = 2x + 4$.

Solution.

$$f(f(x)) = \frac{1}{f(x)} = \frac{1}{\frac{1}{x}} = x$$
$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{2x+4}$$
$$g(f(x)) = 2f(x) + 4 = \frac{2}{x} + 4$$
$$g(g(x)) = 2g(x) + 4 = 2(2x+4) + 4 = 4x + 12$$

□

7. Find $f \circ g \circ h$ where $f(x) = \sqrt{1-x}, g(x) = 1 - x^2, h(x) = 1 + \sqrt{x}$.

Solution. There are several ways to compute the desired expression. One way is to first find $g \circ h$:

$$g \circ h(x) = g(h(x)) = 1 - (h(x))^2 = 1 - (1 + \sqrt{x})^2$$

Then substitute into $f \circ g \circ h$:

$$\begin{aligned} f \circ g \circ h(x) = f(g(h(x))) &= \sqrt{1 - g(h(x))} = \sqrt{1 - [1 - (1 + \sqrt{x})^2]} \\ &= \sqrt{1 - 1 + (1 + \sqrt{x})^2} = 1 + \sqrt{x} \end{aligned}$$

Another way is to compute $f \circ g$:

$$f \circ g(x) = f(g(x)) = \sqrt{1 - g(x)} = \sqrt{1 - (1 - x^2)} = \sqrt{x^2} = x$$

Then substitute h in to find $f \circ g \circ h$:

$$f \circ g \circ h(x) = f(g(h(x)) = h(x) = 1 + \sqrt{x}.$$

□

8. Suppose the graph of f is given. Describe how the following functions transform the graph of f:

   (a) $f(\frac{1}{4}x)$

   Solution. $\frac{1}{4}$ occurs inside the function, hence it is a horizontal stretch where we divide the x values by $\frac{1}{4}$ (or multiply x values by 4). □

   (b) $-f(2x)$

   Solution. The -1 on the outside of the function implies there is a vertical reflection. The 2 on the inside of the function implies there is a horizontal compression where we divide the x values by 2. □

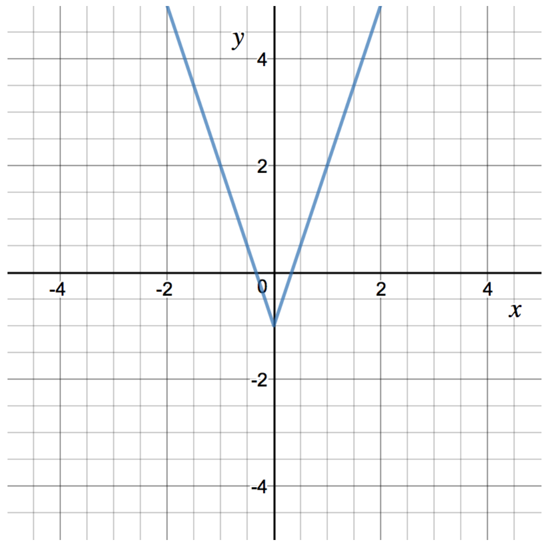
   (c) $f(x - 4) + \frac{3}{4}$

   Proof. The -4 on the inside implies there is a horizontal shift where we will shift right 4 units. The $\frac{3}{4}$ on the outside of the function implies there is a vertical shift where we will shift up $\frac{3}{4}$ units. □

9. Sketch the graphs of the following functions:

   (a) $f(x) = 3|x| - 1$

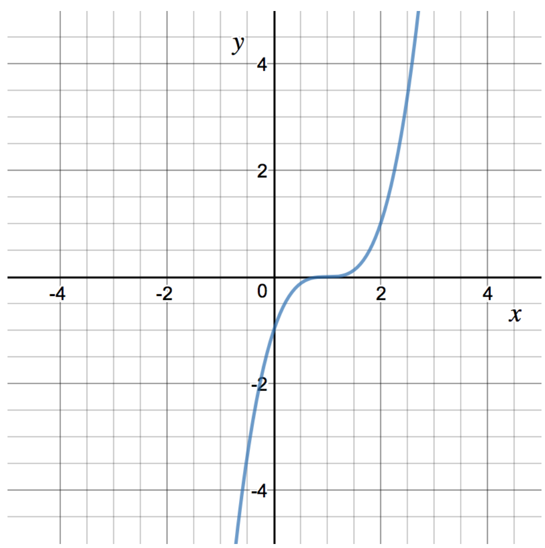
   Solution. The sketch should be $|x|$ vertically stretched by 3 (multiply y values by 3) and shifted down 1. See the plot below.



□

(b) $f(x) = (x-1)^3$

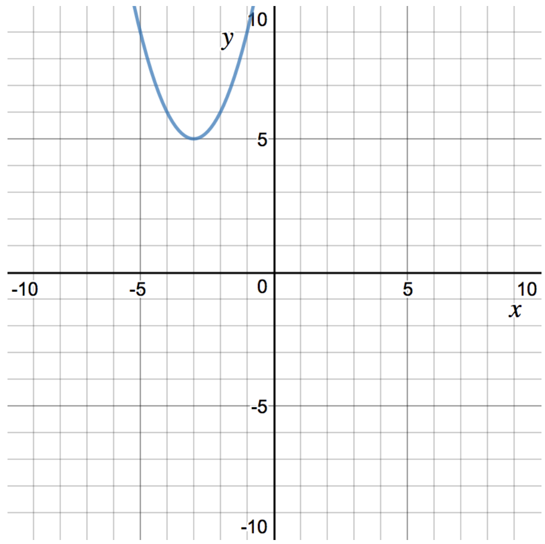
Solution. The sketch should be x^3 shifted right 1. See the plot below.



□

(c) $f(x) = (x+3)^2 + 5$

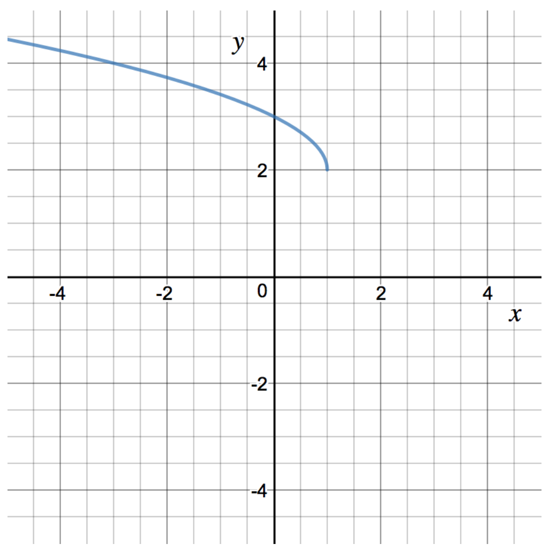
Solution. The sketch should be x^2 shifted left 3 and up 5. See the plot below.



□

(d) $f(x) = 2 + \sqrt{-x+1}$

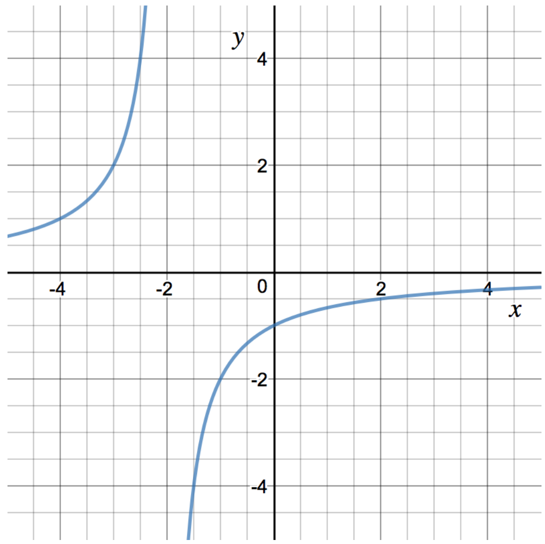
Solution. Rewrite in the form $f(x) = \sqrt{-(x-1)} + 2$. The sketch should be $\sqrt{x}$ reflected horizontally (about the y-axis, multiply x values by -1), shifted right 1 and up 2. See the plot below.



□

(e) $f(x) = \dfrac{-2}{x+2}$

Solution. The sketch should be $\frac{1}{x}$ reflected vertically (about the x-axis, multiply y values by -1), stretched vertically by 2 (multiply y values by 2), shifted left 2. See the plot below.



□

10. For each of the following, determine if f is even, odd, or neither:

(a) $f(x) = x^5 + x^{-3}$

Solution. Calculate $f(-x)$:

$$f(-x) = (-x)^5 + (-x)^{-3} = -x^5 - x^{-3}$$

This is not the same as $f(x)$, so f is not even. Next calculate $-f(x)$:

$$-f(x) = -(x^5 + x^{-3}) = -x^5 - x^{-3}$$

This is the same as $f(-x)$, so f is odd. □

(b) $f(x) = 1 - x^4$

Solution. Calculate $f(-x)$:

$$f(-x) = 1 - (-x)^4 = 1 - x^4$$

This is the same as $f(x)$, so f is even. □

(c) $f(x) = 2x^5 - 3x^2 + 2$

Solution. Calculate $f(-x)$:

$$f(-x) = 2(-x)^5 - 3(-x)^2 + 2 = -2x^5 - 3x^2 + 2$$

This is not the same as $f(x)$, so f is not even. Next calculate $-f(x)$:

$$-f(x) = -(2x^5 - 3x^2 + 2) = -2x^5 + 3x^2 - 2$$

This not the same as $f(-x)$, so f is not odd. Thus f is neither. □

(d) $f(x) = \dfrac{1}{x+2}$

Solution. Calculate $f(-x)$:

$$f(-x) = \frac{1}{-x+2} = -\frac{1}{x-2}$$

This not the same as $f(x)$, so f is not even. Next calculate $-f(x)$:

$$-f(x) = -\frac{1}{x+2}$$

This is not the same as $f(-x)$, so f is not odd. Thus f is neither. □

11. If $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{1}{x} + 1$, verify f and g are inverses of each other. (Don't calculate the inverse directly.)

Solution. We need to show $f(g(x)) = x$, $g(f(x)) = x$.

$$f(g(x)) = \frac{1}{g(x)-1} = \frac{1}{\frac{1}{x}+1-1} = x$$

$$g(f(x)) = \frac{1}{f(x)} + 1 = \frac{1}{\frac{1}{x-1}} + 1 = x - 1 + 1 = x$$

Thus f and g are inverses of each other. □

12. Find the inverse of the following functions:

(a) $f(x) = \sqrt{2x-1}$

Solution. Set $y = f(x)$:

$$y = \sqrt{2x-1}$$

Swap x and y:

$$x = \sqrt{2y-1}$$

Solve for y:

$$x^2 = 2y - 1 \quad \Rightarrow \quad 2y = x^2 + 1 \quad \Rightarrow \quad y = \frac{1}{2}x^2 + \frac{1}{2}$$

Thus $f^{-1}(x) = \frac{1}{2}x^2 + \frac{1}{2}$. □

(b) $g(x) = \dfrac{1}{x+2}$

Solution. Set $y = g(x)$:

$$y = \frac{1}{x+2}$$

Swap x and y:

$$x = \frac{1}{y+2}$$

Solve for y:

$$x(y+2) = 1 \quad \Rightarrow \quad xy + 2x = 1 \quad \Rightarrow \quad xy = 1 - 2x \quad \Rightarrow \quad y = \frac{1-2x}{x} = \frac{1}{x} - 2$$

Thus $g^{-1}(x) = \frac{1}{x} - 2$. $\square$