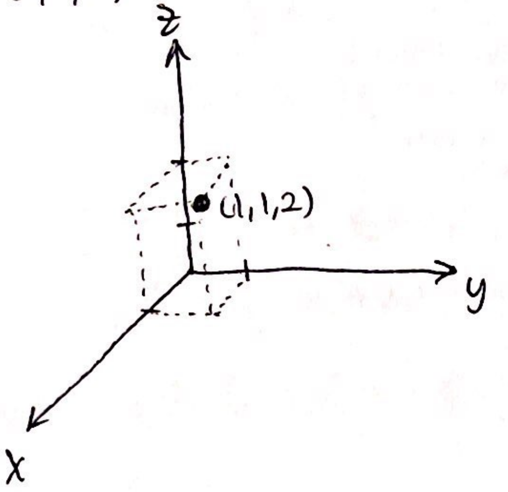
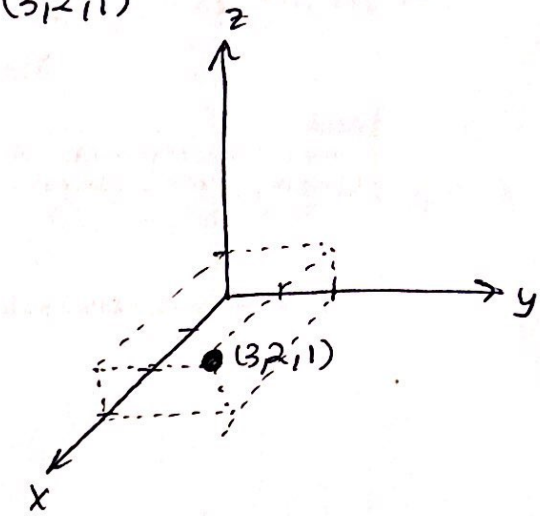
Week 1 Solutions

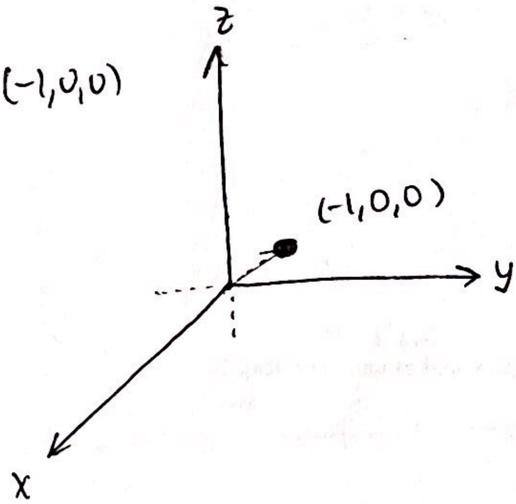
① (a) $(1,1,2)$



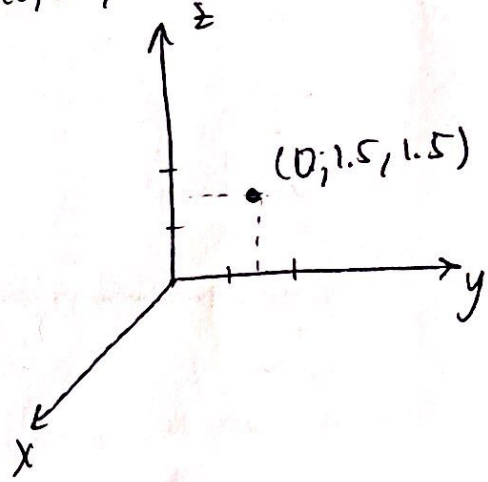
(b) $(3,2,1)$



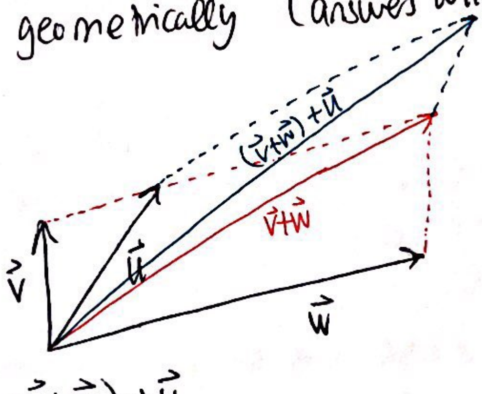
(c) $(-1,0,0)$



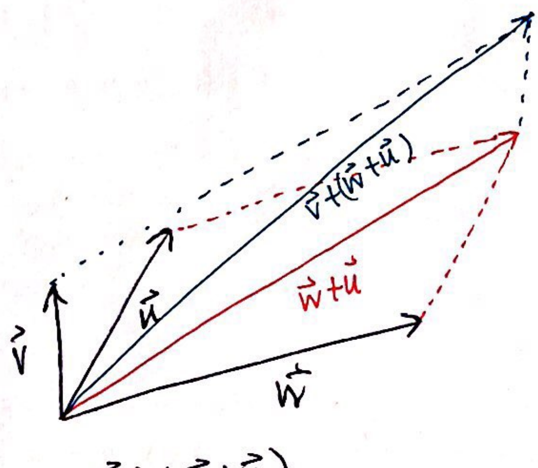
(d) $(0,1.5,1.5)$



② geometrically (answers will vary)



$(\vec{v}+\vec{w})+\vec{u}$



$\vec{v}+(\vec{w}+\vec{u})$

(Not drawn perfectly to scale, but the blue vectors are the same.)

② algebraically

let $\vec{v} = \langle v_1, v_2 \rangle$, $\vec{u} = \langle u_1, u_2 \rangle$, $\vec{w} = \langle w_1, w_2 \rangle$

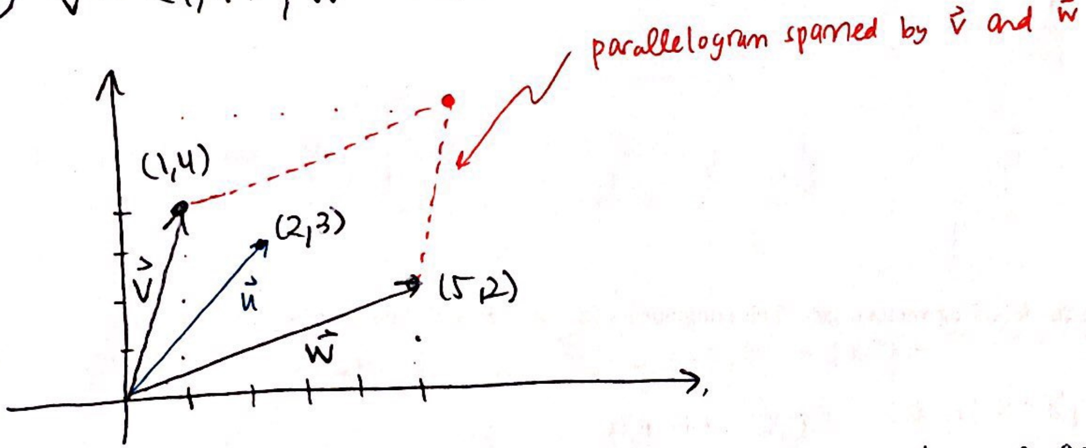
$$\vec{v} + (\vec{w} + \vec{u}) = \vec{v} + (\langle w_1 + u_1, w_2 + u_2 \rangle) = \langle v_1, v_2 \rangle + \langle w_1 + u_1, w_2 + u_2 \rangle$$
$$= \langle v_1 + w_1 + u_1, v_2 + w_2 + u_2 \rangle$$

$$(\vec{v} + \vec{w}) + \vec{u} = (\langle v_1 + w_1, v_2 + w_2 \rangle) + \langle u_1, u_2 \rangle = \langle v_1 + w_1, v_2 + w_2 \rangle + \langle u_1, u_2 \rangle$$
$$= \langle v_1 + w_1 + u_1, v_2 + w_2 + u_2 \rangle$$

same

Same, hence $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{v})$

③ $\vec{v} = \langle 1, 4 \rangle$, $\vec{w} = \langle 5, 2 \rangle$



" $\vec{u}$ linear combination of $\vec{v}, \vec{w}$ " means to find scalars a and b so that

$\vec{u} = a\vec{v} + b\vec{w} \Rightarrow$
$$\langle 2, 3 \rangle = a \langle 1, 4 \rangle + b \langle 5, 2 \rangle$$
$$\langle 2, 3 \rangle = \langle a, 4a \rangle + \langle 5b, 2b \rangle$$
$$\langle 2, 3 \rangle = \langle a + 5b, 4a + 2b \rangle$$

set components equal: (1) $2 = a + 5b$ — system of equations two equations, two unknowns
(2) $3 = 4a + 2b$

Multiply (1) by -4 and add to (2):
$$-8 = -4a - 20b$$
$$3 = 4a + 2b$$
$$\overline{-5 = -18b} \qquad b = 5/18$$

③ continued

$$2 = a + 5\left(\frac{5}{18}\right)$$

$$2 = a + \frac{25}{18}$$

$$2 - \frac{25}{18} = a$$

$$\frac{36-25}{18} = a$$

$$a = \frac{11}{18}$$

Check: $a\langle 1,4\rangle + b\langle 5,2\rangle = \frac{11}{18}\langle 1,4\rangle + \frac{5}{18}\langle 5,2\rangle$

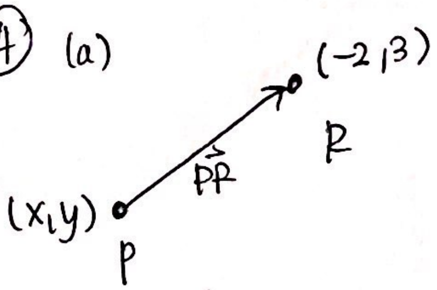
$$= \left\langle \frac{11}{18}, \frac{44}{18}\right\rangle + \left\langle \frac{25}{18}, \frac{10}{18}\right\rangle$$

$$= \left\langle \frac{36}{18}, \frac{54}{18}\right\rangle$$

$$= \langle 2,3\rangle \checkmark$$

So the solution is $\boxed{\vec{u} = \frac{11}{18}\vec{v} + \frac{5}{18}\vec{w}}$

④ (a)



$\vec{PR} = \langle -2,3\rangle - \langle x,y\rangle = \langle -2-x, 3-y\rangle$

Want $\vec{PR} = \langle -2,3\rangle \Rightarrow \langle -2-x, 3-y\rangle = \langle -2,3\rangle$

$\Rightarrow \begin{array}{l} -2-x = -2 \\ 3-y = 3 \end{array} \Rightarrow \begin{array}{l} x = 0 \\ y = 0 \end{array}$

$\boxed{P = (0,0)}$

(b)

$\vec{RQ} = \langle x,y\rangle - \langle -2,3\rangle = \langle x+2, y-3\rangle$

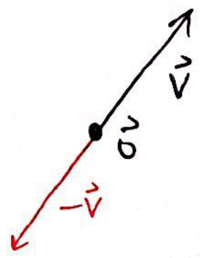
Want $\vec{RQ} = \langle -2,3\rangle \Rightarrow \langle -2,3\rangle = \langle x+2, y-3\rangle$

$\Rightarrow \begin{array}{l} -2 = x+2 \\ 3 = y-3 \end{array} \Rightarrow \begin{array}{l} x = -4 \\ y = 6 \end{array}$ $\boxed{Q = (-4,6)}$

Scanned with CamScanner

⑤ geometrically: (answers will vary)

one way:



diagonal of this "parallelogram" is $\vec{0}$.

another way: add tip to tail

$\Rightarrow$ • $\vec{0}$ $\vec{0}$ vector

algebraically:

let $\vec{v} = \langle v_1, v_2 \rangle$

$$\vec{v} + (-1)\vec{v} = \langle v_1, v_2 \rangle + (-1)\langle v_1, v_2 \rangle = \langle v_1 - v_1, v_2 - v_2 \rangle = \langle 0, 0 \rangle = \vec{0}.$$