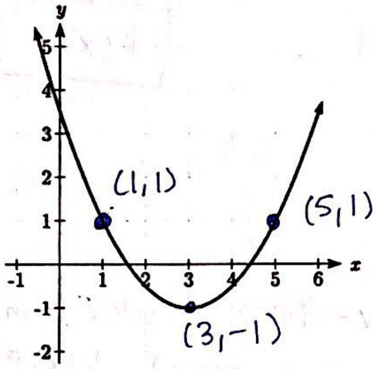
KEY

Math 1 Worksheet for 3.2 and 3.3

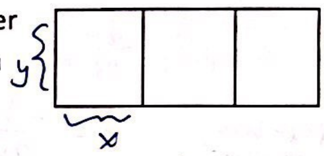
1.Write the equation for this quadratic function.



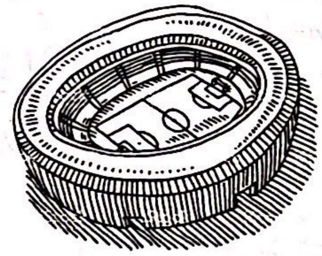
2. Rewrite the quadratic function into vertex form.

$k(x) = 3x^2 - 6x - 9$

3. A farmer wishes to enclose three pens with fencing, as shown. If the farmer has 400 feet of fencing to work with, what dimensions will maximize the area enclosed?



4. A soccer stadium holds 62,000 spectators. With a ticket price of $10, the average attendance has been 24,000. When the price dropped to $8, the average attendance rose to 34,000. Assuming that attendance is linearly related to ticket price, what ticket price would maximize revenue?

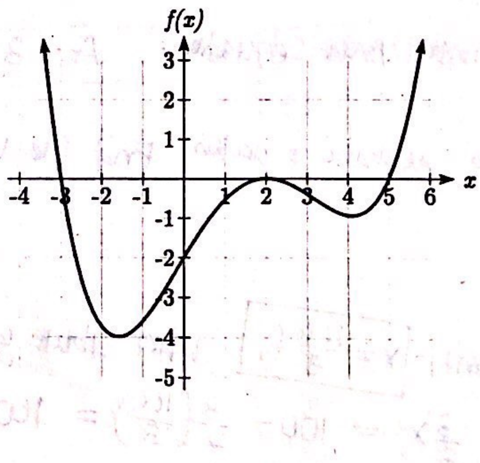


revenue = (profit) x (# of items sold)

5. Find the horizontal intercepts of the polynomial $f(x) = x^6 - 3x^4 + 2x^2$.

6. Find the domain of the function $v(t) = \sqrt{6 - 5t - t^2}$.

7. Write a formula for the polynomial function graphed here.



① $f(x) = a(x-h)^2+k$ vertex is at $(3,-1)$ $\Rightarrow$ $h=3$, $k=-1$

$f(x) = a(x-3)^2 - 1$ Another pt on the graph is $(1,1)$
$x\ f(x)$

$1 = a(1-3)^2-1$, solve for a

$1 = a(-2)^2 - 1$ $\boxed{f(x) = \frac{1}{2}(x-3)^2-1}$

$1 = 4a-1$

$2 = 4a$

$a = 1/2$

② $k(x) = 3x^2-6x-9$ vertex form means $k(x) = a(x-h)^2+k$

one way (completing the square) $k(x) = 3(x^2-2x)-9 = 3(x^2-2x+(-1)^2-(-1)^2)-9$

$-\frac{2}{2} = -1$

$= 3(x^2-2x+1) - 3(-1)^2-9$

$= 3(x-1)(x-1) - 3 - 9 = \boxed{3(x-1)^2-12}$

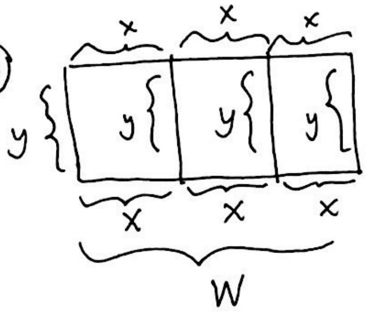
another way $h = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{2(3)} = 1$

$a=3, b=-6, c=9$

$k = k\left(\frac{-b}{2a}\right) = k(1) = 3(1)^2-6(1)-9 = 3-6-9 = -12$

so $k(x) = \boxed{3(x-1)^2-12}$

③



Perimeter is $P = 4y+6x$, but we also know P is 400ft

$\Rightarrow$ $400 = 4y+6x$

Area of each small box is xy, so area of all three is

$A = 3xy$.

solve for x and y using Perimeter equation: $400 = 4y+6x$ (I will solve for y)

$4y = 400-6x$

$y = \frac{400-6x}{4} \Rightarrow y = \frac{400}{4} - \frac{6x}{4} = 100 - \frac{3}{2}x$

Substitute into Area equation: $A = 3x\left(100-\frac{3}{2}x\right) = 300x - \frac{9}{2}x^2$

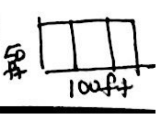
this is a quadratic equation. Find the vertex: $\frac{-b}{2a} = \frac{-300}{2(-\frac{9}{2})} = \frac{-300}{-9} = \frac{300}{9} = \frac{100}{3}$

This is the value for x that maximizes A

so we want $\boxed{x = \frac{100}{3}\text{ ft}}$ What about y? Plug x in to find y.

$y = 100 - \frac{3}{2}x = 100 - \frac{3}{2}\left(\frac{100}{3}\right) = 100 - \frac{100}{2} = 100-50 = 50\text{ft}$

so entire width is $3x$
$= 3\left(\frac{100}{3}\right) = 100$ ft



max capacity 62,000 (Revenue = profit x # items sold)

~~max~~ ticket \$10 ⇒ 24,000

" \$8 ⇒ 34,000

Let x = cost per ticket.

Find linear equation to find attendance. (10, 24,000) and (8, 34,000)

$$m = \frac{34000-24000}{8-10} = \frac{8000}{-2} = -4000$$

$y = -4000x + b$

Plug in (10, 24,000) to find b

$24{,}000 = -4000(10) + b$

$24{,}000 = 40{,}000 + b$

$64{,}000 = b$

Attendance

$y = -4{,}000x + 64{,}000$

Revenue = (cost per ticket) x (# of people who bought tickets)

$= x(-4000x + 64{,}000)$

$= -4000x^2 + 64{,}000x$ quadratic equation

find when max occurs: $x = \frac{-b}{2a} = \frac{-64{,}000}{2(-4{,}000)} = \frac{-64{,}000}{-8{,}000}$

$x = 8$

⇒ $\boxed{\text{ticket should be \$8 to maximize revenue}}$

5. $f(x) = x^6 - 3x^4 + 2x^2 = x^2(x^4 - 3x^2 + 2)$

$x^4 - 3x^2 + 2$ take $u = x^2$

$(x^2)^2 - 3x^2 + 2$

$u^2 - 3u + 2$

$(u-2)(u-1)$

$(x^2-2)(x^2-1)$

So $f(x) = x^2(x^2-2)(x^2-1) = x^2(x-\sqrt{2})(x+\sqrt{2})(x-1)(x+1)$

use $a^2-b^2 = (a-b)(a+b)$

Horizontal intercept means $f(x) = 0$ ⇒ $x^2(x-\sqrt{2})(x+\sqrt{2})(x-1)(x+1) = 0$

$x^2 = 0$ $x - \sqrt{2} = 0$ $x + \sqrt{2} = 0$ $x - 1 = 0$ $x + 1 = 0$

$\boxed{x = 0, \sqrt{2}, -\sqrt{2}, 1, -1}$ or $\boxed{(0,0), (\sqrt{2},0), (-\sqrt{2},0), (1,0), (-1,0)}$

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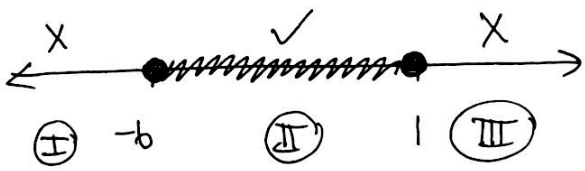
(6) $V(t) = \sqrt{6-5t-t^2}$ need $6-5t-t^2 \geq 0$

$$-(t^2+5t-6) \geq 0$$ multiply by -1 on both sides

$$t^2+5t-6 \leq 0$$

factor $(t+6)(t-1) \leq 0$ set $t+6=0$, $t=-6$ $\quad t-1=0$, $t=1$



make number line, plot $t=-6, t=1$

test points using $(t+6)(t-1) \leq 0$

3 regions : I left of -6, $t=-7 \Rightarrow$ is $(-7+6)(-7-1) \leq 0$?

$-1(-8) \leq 0$? No!

II between -6 and 1, $t=0$: is $(0+6)(0-1) \leq 0$? Yes!

III right of 1, $t=2$: is $(2+6)(2-1) \leq 0$? No!

solution: $\boxed{[-6,1] \text{ or } -6 \leq t \leq 1}$

(7) X-intercepts : $x=-3$ crosses the axis $\rightarrow$ odd multiplicity

$x=2$ touches the axis $\rightarrow$ even multiplicity

$x=5$ crosses the axis $\rightarrow$ odd multiplicity

end behavior $\uparrow$ $\nearrow$ means even degree

$$f(x) = (x-(-3))^1(x-2)^2(x-5)^1$$

$$\boxed{f(x) = (x+3)(x-2)^2(x-5)}$$