Math 1 Practice for Chapter 5 (Week 9) Key

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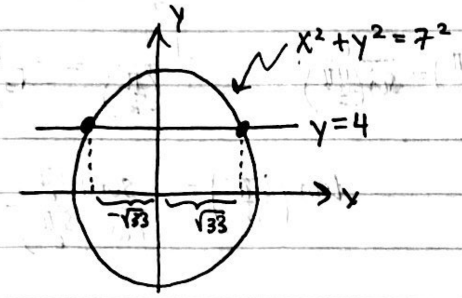
① $(x-1)^2 + (y-(-6))^2 = 4^2$

$(x-1)^2 + (y+6)^2 = 16$

②



Find where y=4 intersects w/ the circle

$x^2 + 4^2 = 49$

$x^2 = 33$

$x = \pm\sqrt{33}$

length $= \sqrt{33} + \sqrt{33} = 2\sqrt{33}$

③ (a) $\frac{\pi}{9} \cdot \frac{180°}{\pi} = \frac{180° \cancel{\pi}}{9\cancel{\pi}} = 20°$

(b) $900° \cdot \frac{\pi}{180°} = \frac{900\pi}{180} = 5\pi$

④ (a) $\theta = 675°$ subtract by 360° until you get a number between 0° and 360°

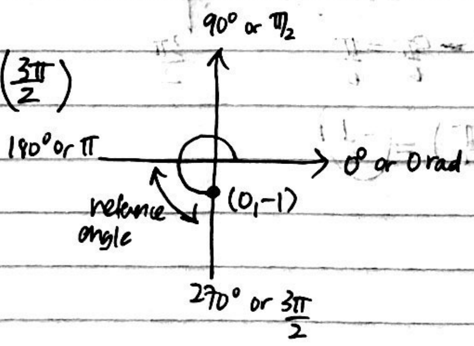
$675° - 360° = \boxed{315°}$

(b) $\theta = \frac{-23\pi}{6}$ add until you get a number between 0 and 2π

$\frac{-23\pi}{6} + 2\pi = \frac{-23\pi}{6} + \frac{12\pi}{6} = \frac{-11\pi}{6}$ not between 0 and 2π

$\frac{-11\pi}{6} + 2\pi = \frac{-11\pi}{6} + \frac{12\pi}{6} = \boxed{\frac{\pi}{6}}$

⑤ (a) $\cos\left(\frac{3\pi}{2}\right)$

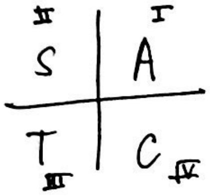


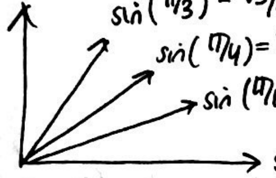
Find the reference angle, i.e. the angle from 180° or π

$\frac{3\pi}{2} - \pi = \frac{\pi}{2}$

So $\cos\left(\frac{3\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

(* Another way: cos is the "x" value of the unit circle at the angle $\frac{3\pi}{2}$, which in this case is 0)





$\sin(\pi/2) = 1$, $\cos \pi/2 = 0$

$\sin(\pi/3) = \sqrt{3}/2$, $\cos(\pi/3) = 1/2$

$\sin(\pi/4) = \sqrt{2}/2 = \cos(\pi/4)$

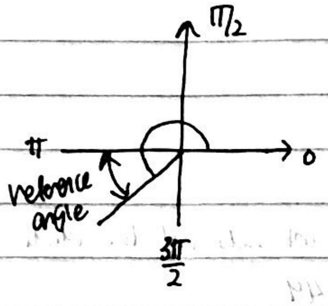
$\sin(\pi/6) = 1/2$, $\cos(\pi/6) = \sqrt{3}/2$

$\sin 0 = 0$, $\cos 0 = 1$

(5) (b) $\csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin\left(\frac{4\pi}{3}\right)}$

Need 2 things: (1) quadrant that $\frac{4\pi}{3}$ is in

(2) reference angle

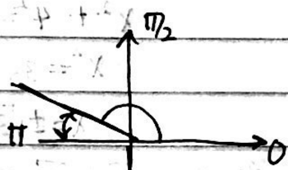


quadrant III, sin is negative

reference angle $\frac{4\pi}{3} - \frac{\pi \cdot 3}{3} = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

Therefore $\csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin\left(\frac{4\pi}{3}\right)} = \frac{1}{-\sin\left(\frac{\pi}{3}\right)} = \frac{1}{-\sqrt{3}/2} = \frac{-2}{\sqrt{3}}$

(c) $\tan\left(\frac{5\pi}{6}\right)$



$\tan\left(\frac{5\pi}{6}\right) = \frac{\sin\left(\frac{5\pi}{6}\right)}{\cos\left(\frac{5\pi}{6}\right)}$

$\frac{5\pi}{6}$ is in quadrant II $\rightarrow$ sin is positive, cos is negative

reference angle is $\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

Therefore $\tan(5\pi/6) = \frac{\sin(5\pi/6)}{\cos(5\pi/6)} = \frac{\sin(\pi/6)}{-\cos(\pi/6)} = \frac{1/2}{-\sqrt{3}/2} = \frac{1}{2} \cdot \frac{-2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

(d) $\cos(6\pi)$

Find an angle α so that $0 \le \alpha < 2\pi$ (i.e. "coterminal")

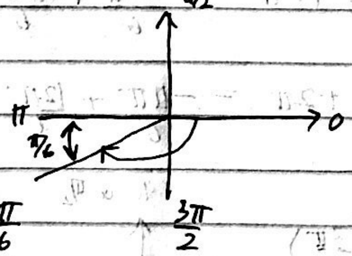
$6\pi - 2\pi = 4\pi$ keep subtracting by 2π

$4\pi - 2\pi = 2\pi$

$2\pi - 2\pi = 0$ ✓ so $\cos 6\pi = \cos 0 = 1$

(e) $\sin\left(\frac{-5\pi}{6}\right)$

negative means clockwise

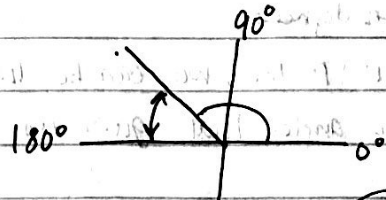


quadrant III, sin is negative

reference angle $\pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

Therefore $\sin\left(\frac{-5\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = \frac{-1}{2}$

⑤ (f) $\cos(135^\circ)$

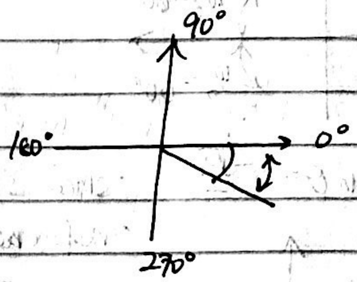


quadrant II, cos is negative

reference angle $180^\circ - 135^\circ = 45^\circ$

Therefore $\cos(135^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

(g) $\cot(-45^\circ) = \dfrac{\cos(-45^\circ)}{\sin(-45^\circ)}$



quadrant IV, cos positive, sin negative

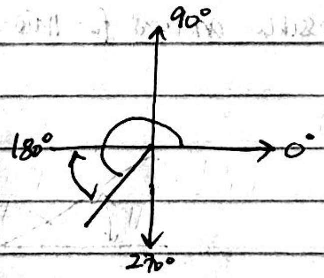
reference angle $0^\circ - (-45^\circ) = 45^\circ$

Therefore

$$\cot(-45^\circ) = \frac{\cos(-45^\circ)}{\sin(-45^\circ)} = \frac{\cos(45^\circ)}{-\sin(45^\circ)} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -\frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = -1$$

(Another way: Find $\tan(-45^\circ) = \dfrac{\sin(-45^\circ)}{\cos(-45^\circ)}$, then use $\cot(-45^\circ) = \dfrac{1}{\tan(-45^\circ)}$)

(h) $\sec 210^\circ = \dfrac{1}{\cos(210^\circ)}$

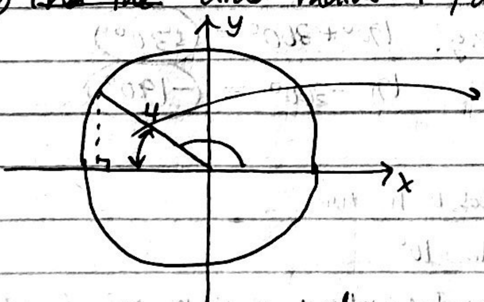


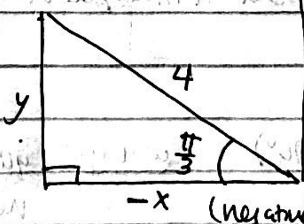
quadrant III, cos negative

reference angle $210^\circ - 180^\circ = 30^\circ$

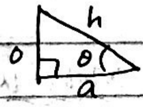
Therefore $\sec 210^\circ = \dfrac{1}{\cos(210^\circ)} = \dfrac{1}{-\cos 30^\circ} = \dfrac{1}{-\sqrt{3}/2} = 1 \cdot \dfrac{-2}{\sqrt{3}} = \dfrac{-2}{\sqrt{3}}$

⑥ ~~Find the~~ circle radius 4, angle $\frac{2\pi}{3}$





(negative b.c. in quadrant II)



"soh cah toa"

$\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$, $\tan\theta = \frac{o}{a}$

reference angle $= \pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$

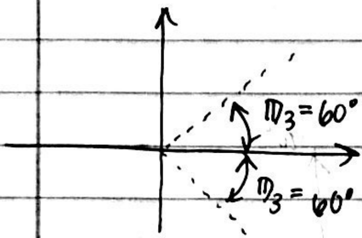
$\sin\left(\frac{\pi}{3}\right) = \frac{y}{4} \Rightarrow y = 4\sin\left(\frac{\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$ $\boxed{(-2, 2\sqrt{3})}$

$\cos\left(\frac{\pi}{3}\right) = \frac{-x}{4} \Rightarrow x = -4\cos\left(\frac{\pi}{3}\right) = -4\left(\frac{1}{2}\right) = -2$

* oops, for 7a use degrees

⑦ (a) $\cos\theta = \frac{1}{2}$

- since cos is positive, we can be in either quadrants I or IV
- the reference angle that gives us $\cos\theta = \frac{1}{2}$ is $\theta = \frac{\pi}{3}$. (or $60°$)



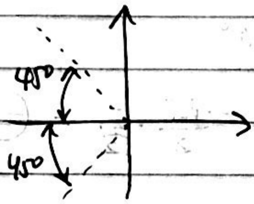
So the angles are $\frac{\pi}{3}$ and $2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

$\Rightarrow 60°$ $\quad 360° - 60° = 300°$

(b) $\sin\theta = \frac{-\sqrt{2}}{2}$

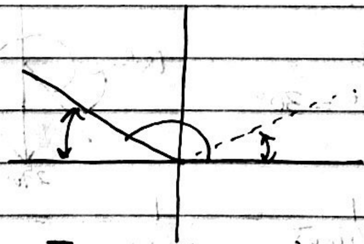
- since sin is negative, we can be in either quadrants II or III
- reference angle is $45°$

So the angles are $180° - 45° = 135°$ and $180° + 45° = 225°$



⑧ Many possible answers for this problem

(a) $\sin(170°)$



one way: positive because quadrant II

reference angle $180° - 170° = 10°$

pick another quadrant w/ positive sin: (i.e. quadrant I) ~~$180°$~~ $10°$

another way: add a multiple of $360°$ e.g. $170° + 360° = 530°$

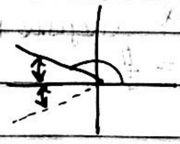
$170° - 360° = -190°$

(b) $\cos(170°)$ one way: quadrant II, cos is negative

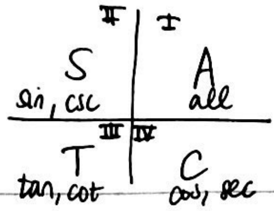
reference angle $10°$

pick another quadrant where cos is negative (quadrant III)

$\rightarrow 190°$



another way: add/subtract $360° \rightarrow 530°, -190°, \ldots$



(9) (a) $\sin\beta > 0 \rightarrow$ quadrant I or II $\Rightarrow$ quadrant I

$\cot\beta > 0 \rightarrow$ quadrant I or III

(b) $\csc\beta > 0 \rightarrow$ quadrant I or II — quadrant II

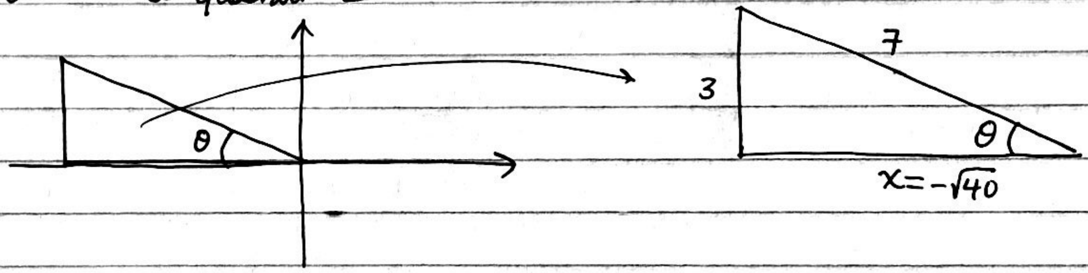
$\sec\beta < 0 \rightarrow$ quadrant II or III

(c) $\tan\beta < 0 \rightarrow$ quadrant II or IV — quadrant IV

$\cos\beta > 0 \rightarrow$ quadrant I or IV

(10) $\sin\theta = \frac{3}{7}$ (positive so quadrant I or II) use "soh cah toa"

$\tan\theta < 0 \longrightarrow$ so quadrant II



pythagorean theorem
$$x^2 + 3^2 = 7^2$$
$$x^2 + 9 = 49$$
$$x^2 = 40$$
$$x = \pm\sqrt{40}$$ (choose negative one b.c. quadrant II)

$\cos\theta = \frac{-\sqrt{40}}{7}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-\sqrt{40}}{7}} = \frac{-7}{\sqrt{40}}$

$\tan\theta = \frac{3}{-\sqrt{40}}$ $\quad$ $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{7}} = \frac{7}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{-\sqrt{40}}} = \frac{-\sqrt{40}}{3}$

(11) $$\frac{\sec^2\theta \sin\theta}{\tan\theta} = \frac{\frac{1}{\cos^2\theta}\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\sin\theta}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{\cancel{\sin\theta}}{\cos^2\theta}\cdot\frac{\cos\theta}{\cancel{\sin\theta}} = \frac{\cos\theta}{\cos^2\theta} = \frac{1}{\cos\theta} = \sec\theta$$