

Math 32B Practice Problems II

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1. Evaluate $\iint_R xy dA$ where R is the region in the first quadrant bounded by the lines $y = x$, $y = 3x$ and the hyperbolas $xy = 1$, $xy = 3$. Use the transformation $y = u/v$, $y = v$.
2. Determine whether or not \mathbf{F} is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.
 - (a) $\mathbf{F}(x, y) = e^x \cos y \mathbf{i} + e^x \sin y \mathbf{j}$
 - (b) $\mathbf{F}(x, y) = (\ln y + 2xy^3) \mathbf{i} + \left(3x^2y^2 + \frac{x}{y}\right) \mathbf{j}$
3. Evaluate the line integral $\int_C xyz ds$ where C is the curve parametrized by $x = 2 \sin t$, $y = t$, $z = -2 \cos t$, $0 \leq t \leq \pi$.
4. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given by the vector function $\mathbf{r}(t) = t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$, $0 \leq t \leq \pi$ and $\mathbf{F} = z\mathbf{i} + y\mathbf{j} - x\mathbf{k}$.
5. Let $\mathbf{F}(x, y, z) = y^2 \cos z \mathbf{i} + 2xy \cos z \mathbf{j} - xy^2 \sin z \mathbf{k}$, C be the curve parametrized by $\mathbf{r}(t) = t^2\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq \pi$.
 - (a) Show that \mathbf{F} is conservative.
 - (b) Find a function f such that $\mathbf{F} = \nabla f$.
 - (c) Use (b) to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C .
6. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and S is the boundary of the solid region E enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.