

Math 31B Practice Problems III

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1. Evaluate the following:

$$\begin{array}{lll} \text{(a)} \int \ln x dx & \text{(d)} \int \arctan\left(\frac{1}{x}\right) dx & \text{(g)} \int \frac{10}{(x-1)(x^2+9)} dx \\ \text{(b)} \int x e^{-x} dx & \text{(e)} \int x^3 e^{-x^2} dx & \text{(h)} \int \frac{1}{x^2-16} dx \\ \text{(c)} \int (x^2+1) \cos(4x) dx & \text{(f)} \int \frac{x+1}{x^2-2x+2} dx & \text{(i)} \int \frac{1}{(x+5)^2(x-1)} dx \end{array}$$

2. Determine whether the following improper integrals converge or diverge:

$$\begin{array}{lll} \text{(a)} \int_{-\infty}^{\infty} \frac{1}{x^2} dx & \text{(b)} \int_5^{10} \frac{1}{x-5} dx & \text{(c)} \int_2^{\infty} \frac{1}{x(\ln x)^3} dx \end{array}$$

3. Determine the radius and interval of convergence of the following power series:

$$\begin{array}{lll} \text{(a)} \sum_{n=100}^{\infty} \frac{x^n}{n!} & \text{(b)} \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n} & \text{(c)} \sum_{n=1}^{\infty} n!(2x-1)^n \end{array}$$

4. Use the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$, to find the power series of the following functions:

$$\begin{array}{ll} \text{(a)} f(x) = \frac{1}{x^7-1} & \text{(b)} f(x) = \frac{x^3}{x+2} \end{array}$$

5. Consider the function $f(x) = \frac{1}{x}, a = -3$.

- Write the expression for the error $|f(-3.1) - T_n(-3.1)|$ where T_n denote the n th Taylor polynomial.
- Find the Taylor series for $f(x)$ centered about a .

6. Use an appropriate Taylor series to evaluate the following:

$$\begin{array}{lll} \text{(a)} \int e^{x^2} dx & \text{(b)} \int x \cos(x^3) dx & \text{(c)} \lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3} \end{array}$$