Math 33A Practice Problems I

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1. Circle true (T) or false (F) for each statement below. *Optional challenge: Can you explain/prove why each statement is true or false?*

   (a) T F It is possible for a system of linear equations to have exactly three solutions.

   (b) T F The following system is consistent:

   $$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

   (c) T F The following system has a unique solution:

   $$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_2 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

   (d) T F The vector $(3, -1)$ is a solution to the system $A\mathbf{x} = \mathbf{b}$ where

   $$A = \begin{pmatrix} 1 & 2 \\ -2 & 5 \\ -5 & 6 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 7 \\ 4 \\ -3 \end{pmatrix}$$

   (e) T F Any system with a 3×3 matrix with rank 3 has a unique solution.

   (f) T F Any system with a 3×5 matrix with rank 3 has a unique solution.

   (g) T F Any system with a 4×3 matrix with rank 3 has a unique solution.

   (h) T F For any two matrices A, B, the sum $A + B$ is always defined.

   (i) T F All transformations are linear transformations.

   (j) T F Every transformation $T(x)$ can be written as $T(x) = Ax$ for some matrix A.

   (k) T F Every linear transformation $T(x)$ can be written as $T(x) = Ax$ for some matrix A.

   (l) T F The transformation $T(\mathbf{x}) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x}$ projects $\mathbf{x} = (x, y)$ onto the y-axis.

   (m) T F If A and B are square matrices, then $AB = BA$.

   (n) T F For any two matrices A, B, the product AB is always defined.

   (o) T F All square matrices have an inverse.

   (p) T F If A and B are invertible matrices then $(AB)^{-1} = A^{-1}B^{-1}$.

   (q) T F If A and B are invertible matrices then $(AB)^{-1} = B^{-1}A^{-1}$.

2. Consider the system

$$\begin{aligned} x_1 + \quad\quad + x_3 &= 2 \\ + x_2 + \alpha x_3 &= 0 \\ x_1 + 2x_2 + 13x_3 &= 0 \end{aligned}$$

   (a) For what values of α is the system

      i. Inconsistent?

      ii. Consistent?

   (b) Is it possible to find a value of α such that the system has infinitely many solutions? Why or why not?

   (c) For $\alpha = 7$, find the solution of the system using Gauss-Jordan elimination.

3. Let $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$.

   (a) Find $3\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3$.

   (b) Can $\mathbf{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 9 \end{pmatrix}$ be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_1, \mathbf{v}_3$? If so, find it.

   (c) Can $\mathbf{b}_2 = \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$ be written as a linear combination of $\mathbf{v}_1, \mathbf{v}_1, \mathbf{v}_3$? If so, find it.

4. Let T be the transformation defined by the formula

$$T(x_1, x_2, x_3) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_3)$$

   (a) Show T is linear.

   (b) Find a matrix A for the linear transformation such that $T(\mathbf{x}) = A\mathbf{x}$.

   (c) Find $T(2, -1, 0)$.

5. The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that maps $(1, 2)$ to $(-1, 1)$ and $(0, -1)$ to $(2, -1)$ will map $(1, 1)$ to what value?

6. Let $T(\mathbf{x})$ be the linear transformation that reflects $\mathbf{x}$ about the plane $-x_1 + 3x_2 + 2x_3 = 0$.

   (a) Find $T(1, 4, 0)$.

   (b) Find the matrix representation for $T(\mathbf{x})$.

   (c) Without calculating directly, what is $T \circ T(\mathbf{x})$?

   (d) Without calculating directly, what is $T \circ T \circ T(\mathbf{x})$?

7. Let $B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$. Compute the following:

(a) $B^2 - 2B + I$

(b) B^{-3}

8. If the matrices

$$\begin{pmatrix} 3 & -2 & -1 \\ -1 & 1 & 1 \\ 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix}$$

are inverse of each other, what is the value of c?

9. (a) Calculate the inverse of

$$\begin{pmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{pmatrix}$$

(b) Use what you found in part (a) to solve the system

$$\begin{aligned} 3x_1 + 4x_2 - x_3 &= 1 \\ x_1 + \quad +3x_3 &= 0 \\ 2x_1 + 5x_2 - 4x_3 &= 2 \end{aligned}$$