Math 33A − Week 3

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April 16, 2019

Name: KEY

1. Let $(r \times s)$ denote a matrix with size $r \times s$. State whether each product is defined. If it is defined, state the size of the resulting matrix.

   (a) $(2 \times 3)(3 \times 5)$ defined, 2x5

   (b) $(2 \times 2)(2 \times 3)$ defined, 2x3

   (c) $(3 \times 4)(3 \times 4)$ undefined

   (d) $(4 \times 1)(1 \times 3)$ defined, 4x3

   (e) $(1 \times 2)(4 \times 1)$ undefined

   (f) $(5 \times 2)(2 \times 2)$ defined, 5x2

2. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix}$

   (a) Compute AB and BA.

$$AB = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 0+2\cdot 7+3\cdot 2 & 1\cdot 1+2(-1)+3\cdot 4 \\ 4\cdot 0+5\cdot 7+6\cdot 2 & 4\cdot 1+5(-1)+6\cdot 4 \end{pmatrix} = \begin{pmatrix} 0+14+6 & 1-2+12 \\ 0+35+12 & 4-5+24 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 11 \\ 47 & 23 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 1 \\ 7 & -1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 0\cdot 1+1\cdot 4 & 0\cdot 2+1\cdot 5 & 0\cdot 3+1\cdot 6 \\ 7\cdot 1-1\cdot 4 & 7\cdot 2-1\cdot 5 & 7\cdot 3-1\cdot 6 \\ 2\cdot 1+4\cdot 4 & 2\cdot 2+4\cdot 5 & 2\cdot 3+4\cdot 6 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 3 & 9 & 15 \\ 18 & 24 & 30 \end{pmatrix}$$

   (b) Does $AB = BA$?

   No

3. (a) Using the formula

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix},$$

calculate A^{-1} if $A = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$.

$$\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{3\cdot 2-(-1)(-5)}\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \frac{1}{6-5}\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}}$$

(b) Verify your matrix in (a) is the inverse of A by computing AA^{-1} and $A^{-1}A$.

$$AA^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3\cdot 2-5\cdot 1 & 3\cdot 5-5\cdot 3 \\ -1\cdot 2+2\cdot 1 & -1\cdot 5+2\cdot 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \checkmark$$

$$A^{-1}A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2\cdot 3-1\cdot 5 & 2(-5)+2\cdot 5 \\ 1\cdot 3-1\cdot 3 & 1(-5)+2\cdot 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \checkmark$$