Solutions to Final Practice T/F Questions

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Circle true (T) or false (F) for each statement below. If the statement is false, show why, provide a counterexample, or correct the statement (where possible).

1. F If A and B are square matrices, then $\det(A+B) = \det A + \det B$

   Find a counterexample.

2. F If A and B are square matrices, then $\det(AB) = \det(A)\det(B)$.

   AB might not be defined.

3. T If A and B are square matrices and AB is defined, then $\det(AB) = \det(A)\det(B)$.

4. F All matrices have an inverse.

5. F All square matrices have an inverse.

   Find a counterexample.

6. T All square matrices with nonzero determinant have an inverse.

7. F If A has eigenvalue λ, then A^k has eigenvalue $k\lambda$.

   See practice problem #17(a).

8. F For any two matrices A, B, the product AB is always defined.

9. F For any two matrices A, B the sum $A+B$ is always defined.

10. F If A is an $m \times n$ matrix, the Rank-Nullity Theorem states $rank(A) + nullity(A) = m$.

    How can you correct this statement to make it true?

11. F The following system is consistent:

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

12. F It is possible for a system of equations to have exactly two solutions.

    A system can have either no solution, exactly one solution, or infinitely many solutions.

13. T The following system has a unique solution:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned}$$

14. F The vector $(3,-1)$ is a solution to the system $A\mathbf{x}=\mathbf{b}$ where

$$A=\begin{pmatrix}1 & 2\\ -2 & 5\\ -5 & 6\end{pmatrix}, \quad \mathbf{b}=\begin{pmatrix}7\\ 4\\ -3\end{pmatrix}$$

15. F Let $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$. The formula for A^{-1} is

$$A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}a & -b\\ -c & d\end{pmatrix}.$$

How can you correct this statement to make it true?

16. T The general solution of $A\mathbf{x}=\mathbf{b}$ where

$$A=\begin{pmatrix}3 & 5 & -4\\ -3 & -2 & 4\\ 6 & 1 & -8\end{pmatrix}, \quad \mathbf{b}=\begin{pmatrix}7\\ -1\\ -4\end{pmatrix}$$

is $\mathbf{x}=\begin{pmatrix}-1\\ 2\\ 0\end{pmatrix}+\begin{pmatrix}\frac{4}{3}\\ 0\\ 1\end{pmatrix}x_3$.

17. T If A is diagonalizable then there exists a D and invertible P such that $A=PDP^{-1}$.

18. T If A is similar to B, then A^k is similar to B^k.

19. T The set of eigenvalues λ of A are the set of values such that $A-\lambda I$ is not invertible.
Look at $\det(A-\lambda I)=0$.

20. F The following set of vectors are linearly independent:

$$\left\{\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}, \begin{pmatrix}4\\ 5\\ 6\end{pmatrix}, \begin{pmatrix}2\\ 1\\ 0\end{pmatrix}\right\}$$

21. F The set of vectors in the previous problem (#17) span $\mathbb{R}^2$.
Vectors in $\mathbb{R}^3$ cannot span $\mathbb{R}^2$. *What happens in* $\mathbb{R}^3$ *stays in* $\mathbb{R}^3$.

22. T If A is an invertible matrix then $\det(A^{-1})=\dfrac{1}{\det(A)}$.

23. F All transformations are linear transformations.
Find a counterexample (see Midterm 2 Practice Problems # 2(b)(d)).

24. T The transformation defined by $T(\mathbf{x})=3\mathbf{x}$ is a linear transformation.

25. T The matrix representation for the transformation $T(\mathbf{x})=3\mathbf{x}$ is $A=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$.

26. T If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $null(A) = \{\mathbf{0}\}$.

27. F If A and B are invertible matrices then $(AB)^{-1} = A^{-1}B^{-1}$.

28. T If A and B are invertible matrices then $(AB)^{-1} = B^{-1}A^{-1}$.

29. F If the product AB is defined, then $(AB)^T = A^T B^T$.

30. T If the product AB is defined, then $(AB)^T = B^T A^T$.

31. F The set $U = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 : x_1 \geq 0, x_2 \geq 0 \right\}$ is a subspace of $\mathbb{R}^2$.

Show the set is not closed under scalar multiplication.

32. F The rank is the dimension of the null space.

33. T The rank is the dimension of the column space.

34. T The rank is the dimension of the row space.

35. T The rank is the number of pivots of a reduced matrix.

36. T The nullity is the number of free variables of a reduced matrix.

37. F The determinant of a matrix is the product of its diagonal entries.

Find a counterexample.

38. T The determinant of a triangular (upper or lower) matrix is the product of its diagonal entries.

39. T $\begin{vmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{vmatrix} = 20$

40. T If A is a square matrix then $\det A = \det A^T$.

41. T If $T(\mathbf{x}) = A\mathbf{x}$, then the kernel of T is the null space of A.

42. T The pivot columns of a matrix form the column space of that matrix.

43. T The following set of vectors forms a basis for $\mathbb{R}^3$:

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

44. F The dot product of two vectors is always defined.

What if two vectors have different sizes?

45. T The matrix $\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$ has the characteristic equation $(\lambda - 3)(\lambda + 7)$.

46. F All square matrices are diagonalizable.

An $n \times n$ matrix is diagonalizable if and only if it has n linearly independent eigenvectors.

47. F The norm of a vector $\mathbf{u}$ is given by $||\mathbf{u}|| = \mathbf{u} \cdot \mathbf{u}$.

How can you correct this statement to make it true?

48. T The dot product is defined to be $\mathbf{u} \cdot \mathbf{v} = ||\mathbf{u}||||\mathbf{v}|| \cos \theta$.

49. T The matrix $U = \begin{pmatrix} 3/\sqrt{11} & -1/\sqrt{6} & -1/\sqrt{66} \\ 1/\sqrt{11} & 2/\sqrt{6} & -4/\sqrt{66} \\ 1/\sqrt{11} & 1/\sqrt{6} & 7/\sqrt{66} \end{pmatrix}$ is orthogonal.

50. T The set of vectors below forms an orthogonal set:

$$\left\{ \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix} \right\}$$