Midterm 1 Practice Problems

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Name: ______________________________ Score : NA

Directions: You may use one 3" × 5" notecard, however no other outside resources such as books, notes, or calculators are allowed. Write your solutions in your bluebook and clearly mark the problems. If you solve a problem multiple times, cross out the ones you do not want graded, otherwise you will receive little or no partial credit. Unless otherwise specified, numbers included in a solution are not to be approximated, but instead expressed as exact numbers (i.e., in terms of square roots, multiples of π, etc.).

Disclaimer: The content and level of difficulty of these practice questions are not guaranteed to be in correlation with the midterm nor final examination in any form.

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1. Solve the following systems using row operations:

   (a) $x_1 + x_2 + 2x_3 = 8$
   $-x_1 - 2x_2 + 3x_3 = 1$
   $3x_1 - 7x_2 + 4x_3 = 10$

   (b) $2x + 2y + 2z = 0$
   $-2x + 5y + 2z = 1$
   $8x + y + 4z = -1$

   (c) $-2b + 3c = 1$
   $3a + 6b - 3c = -2$
   $6a + 6b + 3c = 5$

2. Let $\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \mathbf{v_2} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \mathbf{v_3} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$.

   (a) Find $3\mathbf{v_1} + 2\mathbf{v_2} - \mathbf{v_3}$.

   (b) Can $\mathbf{b_1} = \begin{pmatrix} 1 \\ 1 \\ 9 \end{pmatrix}$ be written as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$? If so, find it.

   (c) Can $\mathbf{b_2} = \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$ be written as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$? If so, find it.

   (d) Are $\mathbf{b_1}$, $\mathbf{b_2}$ in span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$? Use your results from parts (b) and (c).

   (e) Does $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$?

3. Let $\mathbf{v_1} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$, $\mathbf{v_2} = \begin{pmatrix} 0 \\ -3 \\ 8 \end{pmatrix}$, $\mathbf{v_3} = \begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix}$. Does $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$? Why or why not?

4. Find the general solution of the system whose augmented matrix is given by

   (a) $\left(\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \end{array}\right)$

   (b) $\left(\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array}\right)$

5. Show whether the following sets of vectors in $\mathbb{R}^3$ are linearly independent or linearly dependent.

   (a) $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 10 \\ 2 \end{pmatrix}$

   (b) $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

   (c) $\begin{pmatrix} 8 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

   (d) $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$