Midterm 2 Practice Problems

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Name: ______________________________ Score: NA

Directions: You may use one 3" × 5" notecard, however no other outside resources such as books, notes, or calculators are allowed. Write your solutions in your bluebook and clearly mark the problems. If you solve a problem multiple times, cross out the work you do not want graded, otherwise you will receive little or no partial credit. Unless otherwise specified, numbers included in a solution are not to be approximated, but instead expressed as exact numbers (i.e., in terms of square roots, multiples of π, etc.).

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm nor final examinations in any form.

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1. Let T be the linear transformation defined by the formula

$$T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_3).$$

   (a) Find the standard matrix A for the linear transformation such that $T(\mathbf{x}) = A\mathbf{x}$.
   (b) Find the image of $(x_1, x_2) = (2, -1)$.
   (c) Find the kernel of T (*Hint*: This is the null space of A).
   (d) Find the range of T (*Hint*: This is the column space of A).

2. Determine whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear operator where

   (a) $T(x, y) = (2x + y, x - y)$
   (b) $T(x, y) = (x + 1, y)$
   (c) $T(x, y) = (y, y)$
   (d) $T(x, y) = (\sqrt[3]{x}, \sqrt[3]{y})$

3. Consider the matrices

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}$$

   Compute the following (where possible). If the operation is not defined, explain why.

   (a) $B^2 - 2B + I$.
   (b) $3A^T - C$
   (c) BD
   (d) $(AC)D$

(e) $CB - 2A$

(f) B^{-3}

(g) CC^T

4. Find the inverse of $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

5. Let $G = \begin{pmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{pmatrix}$.

   (a) Find $\det(G)$.

   (b) Does G^{-1} exist? If so, find it.

6. Let $J = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{pmatrix}$.

   (a) Find $\det(J)$.

   (b) Does J^{-1} exist? If so, find it.

7. Let $U = \{(x, y) : x \geq -2, y \leq 1\}$ be a subset of $\mathbb{R}^2$. Is U a subspace of $\mathbb{R}^2$? Why or why not?

8. Let $\mathbf{v}_1 = (1, 2, 1), \mathbf{v}_2 = (2, 9, 0), \mathbf{v}_3 = (3, 3, 4)$. Show that the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\mathbb{R}^3$.

9. Let $K = \begin{pmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{pmatrix}$.

   (a) Find a basis for the column space of K.

   (b) Find a basis for the null space of K.