Math 4A: "Quiz" 1

Week of April 10, 2017

Name: ______________________________ Score: __________/10

Circle which section you attend: M 6pm M 7pm W 4pm W 7pm

Directions: No outside resources such as books, notes, or calculators are allowed.

Disclaimer: The content and level of difficulty of this quiz are not guaranteed to be in correlation with the midterm or final examinations in any form.

For the following problems, consider the system

$$\begin{aligned} x_1 + x_3 &= 2 \\ x_2 + \alpha x_3 &= 0 \\ x_1 + 2x_2 + 13x_3 &= 0 \end{aligned}$$

1. For what values of α is the system
   (a) inconsistent?
   (b) consistent?

   Solution. Use row operations to write the matrix in row echelon form:

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 1 & 2 & 13 & 0 \end{array}\right) \xrightarrow{-R1+R3 \to R3} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 2 & 12 & -2 \end{array}\right) \xrightarrow{-2R2+R3 \to R3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 12-2\alpha & -2 \end{array}\right)$$

   You may have a multiple of the last row, that is okay.

   (a) For the solution to be inconsistent means we need to get a false statement, i.e. row of zeros to the left of the "bar" and a nonzero number to the right of the "bar". Set $12 - 2\alpha = 0$, which yields $\alpha = 6$.

   (b) For the solution to be consistent means we need to get a row of zeros (see the next question) or we need to get a nonzero pivot in the third row. Set $12 - 2\alpha \neq 0$, which yields $\alpha \neq 6$.

   □

2. Is it possible to find a value of α such that the system has infinitely many solutions? Why or why not?

   Solution. No, there is no such α that yields infinitely many solutions. There needs to be a free variable to have infinitely many solutions. x_1 and x_2 are not free, as there are pivots in their respective columns. The only option is to have a missing pivot in the last row to get a row of zeros: $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right)$. However, if $12 - 2\alpha = 0$, then our last row is now $\left(\begin{array}{ccc|c} 0 & 0 & 0 & -2 \end{array}\right)$, which means we have an inconsistent system. We cannot get a row of zeros, therefore we cannot have a free variable, therefore there cannot be infinitely many solutions. □

3. For $\alpha = 7$, find the solution of the system using Gauss-Jordan elimination.

   Solution. We continue row operations on the augmented matrix from Exercise 1:

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & \alpha & 0 \\ 0 & 0 & 12-2\alpha & -2 \end{array}\right) \xrightarrow{\alpha = 7} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & -2 & -2 \end{array}\right) \xrightarrow{-\frac{1}{2}R3 \to R3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-7R3 + R2 \to R2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{R1 - R3 \to R3}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

   The solution to the system is $x_1 = 1, x_2 = -7, x_3 = 1$. □