Quiz 4 Solutions

January 31, 2016

1. Solve the equation $y^{(4)} - 16y = 0$.

   Solution. The characteristic equation to the differential equation is $r^4 - 16 = 0$. This can be factored as
   $$(r^2 - 4)(r^2 + 4) = 0 \quad \Rightarrow \quad (r-2)(r+2)(r^2+4) = 0.$$
   $r^2 + 4 = 0$ has the roots $r = \pm 2i$, therefore the zeros to $r^4 - 16 = 0$ are $r = 2, -2, \pm 2i$. Therefore the general solution to the differential equation is
   $$y = c_1 e^{2x} + c_2 e^{-2x} + e^{0x}(c_3 \cos 2x + c_4 \sin 2x) \quad \Rightarrow \quad y = c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x.$$
   □

2. Solve the equation $\dfrac{d^5y}{dx^5} - 2\dfrac{d^4y}{dx^4} + 5\dfrac{d^3y}{dx^3} = 0$. The characteristic equation to the differential equation is $r^5 - 2r^4 + 5r^3 = 0$. This can be factored as
   $$r^3(r^2 - 2r + 5) = 0.$$
   $r^2 - 2r + 5 = 0$ has the roots $r = 1 \pm 2i$ (use the quadratic equation), therefore the zeros to $r^5 - 2r^4 + 5r^3 = 0$ are $r = 0$ (multiplicity 3) and $r = 1 \pm 2i$. Therefore the general solution to the differential equation is
   $$y = c_1 e^{0x} + c_2 x e^{0x} + c_3 x^2 e^{0x} + e^{1x}(c_4 \cos 2x + c_5 \sin 2x)$$
   $$\Rightarrow \quad y = c_1 + c_2 x + c_3 x^2 + e^x(c_4 \cos 2x + c_5 \sin 2x).$$