Practice Problems III

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1. Solve $\mathbf{x}' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \mathbf{x}$.

2. Solve $\mathbf{x}' = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x}$.

3. Solve $\mathbf{x}' = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 6 & 0 \\ -4 & 0 & 4 \end{pmatrix} \mathbf{x}$.

4. Consider the system $\mathbf{x}' = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix} \mathbf{x}$.
   (a) Find the general solution of the system.
   (b) Sketch a phase plane portrait and classify the system's geometric character and stability behavior.
   (c) Solve the given initial value problem: $\mathbf{x}' = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

5. Consider the system $\mathbf{x}' = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \mathbf{x}$.
   (a) Find the general solution of the system.
   (b) Sketch a phase plane portrait and classify the system's geometric character and stability behavior.
   (c) Solve the given initial value problem: $\mathbf{x}' = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$.

6. Consider the system $\mathbf{x}' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \mathbf{x}$.
   (a) Find the general solution of the system.
   (b) Sketch a phase plane portrait and classify the system's geometric character and stability behavior.
   (c) Solve the given initial value problem: $\mathbf{x}' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

7. Use the method of undetermined coefficients to solve the system $\mathbf{x}' = \begin{pmatrix} 4 & \frac{1}{3} \\ 9 & 6 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -3 \\ 10 \end{pmatrix} e^t$.

8. Consider the autonomous system
$$x' = y - x^2 + 2$$
$$y' = x^2 - xy$$
   (a) Find the fixed points of the system.
   (b) Write the Jacobian J for the system above.
   (c) For each of your fixed points in part (a), evaluate the Jacobian J you found in part (b) and use it to classify the type and stability of that fixed point.