Quiz 1 Solutions

June 23, 2016

1. Find the solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = 3 - 4y \\ y(0) = 1 \end{cases}$$

Solution. You can solve this problem using separation of variables or finding an integrating factor.

(i) Separation of variables: Rewrite as

$$\frac{dy}{3-4y} = dt$$

Integrate both sides:

$$-\frac{1}{4}\ln|3-4y| = t + c_1$$

Solve for y:

$$\begin{aligned} \ln|3-4y| &= -4t + c_2 \\ \pm(3-4y) &= e^{-4t}e^{c_2} \\ 3-4y &= c_3 e^{-4t} \\ y &= \frac{3}{4} - \frac{c_3}{4}e^{-4t} \\ y &= \frac{3}{4} + Ce^{-4t} \end{aligned}$$

When $t = 0, y = 1$:

$$1 = \frac{3}{4} + C \quad \Rightarrow \quad C = \frac{1}{4}.$$

Plug C back in:

$$y = \frac{3}{4} + \frac{1}{4}e^{-4t}.$$

(ii) Integrating factor: Rewrite as

$$\frac{dy}{dt} + 4y = 3$$

Find integrating factor:

$$\mu = e^{\int P(t)dt} = e^{\int 4dt} = e^{4t}$$

Multiply equation by μ:

$$\begin{aligned} e^{4t}\frac{dy}{dt} + 4e^{4t}y &= 3e^{4t} \\ (e^{4t}y)' &= 3e^{4t} \end{aligned}$$

Integrate:

$$\begin{aligned} e^{4t}y &= \int 3e^{4t}dt \\ e^{4t}y &= \frac{3}{4}e^{4t} + C \end{aligned}$$

Solve for y:

$$y = \frac{3}{4} + Ce^{-4t}.$$

When $t = 0, y = 1$:

$$1 = \frac{3}{4} + C \quad \Rightarrow \quad C = \frac{1}{4}.$$

Plug C back in:

$$y = \frac{3}{4} + \frac{1}{4}e^{-4t}.$$

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