

Quiz 4 Solutions

July 10, 2016

1. Consider the initial value problem:

$$(4 - t^2)y' + 2ty = \ln(t + 5), y(-3) = 1.$$

Determine (without solving the problem) the largest interval in which the solution of the IVP is certain to exist.

Solution. Write the equation in standard form:

$$y' + \frac{2t}{4 - t^2}y = \frac{\ln(t + 5)}{4 - t^2}.$$

The domain of $P(t) = \frac{2t}{4 - t^2}$ is all t except $t \neq \pm 2$. Since $\ln(t)$ has the domain $t > 0$, the domain of $f(t) = \frac{\ln(t + 5)}{4 - t^2}$ is all $t > -5$ and $t \neq \pm 2$. The domain of our equation can therefore be written as $(-5, -2) \cup (-2, 2) \cup (2, \infty)$ in interval notation.

The point $y(-3) = 1$ occurs when $x = -3$ which is in the interval $(-5, -2)$. Therefore the Existence and Uniqueness theorem guarantees a unique solution in the interval $(-5, -2)$. \square