Review Quiz Solutions

June 23, 2016

1. Evaluate the following derivatives:

   (a) $\frac{d}{dx}x^{100}$

   Solution. Use the power rule $\frac{d}{dx}x^n = nx^{n-1}$:

   $$\frac{d}{dx}x^{100} = 100x^{99}.$$

   □

   (b) $\frac{d}{dx}(xe^x)$

   Solution. Use the product rule: $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$:

   $$\frac{d}{dx}(xe^x) = x'e^x + x(e^x)' = e^x + xe^x = (1+x)e^x.$$

   □

   (c) $\left(\sin^{-1}(2x)\right)'$

   Solution. Use the chain rule $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$:

   $$\left(\sin^{-1}(2x)\right)' = \frac{1}{\sqrt{1-(2x)^2}} \cdot (2x)' = \frac{2}{\sqrt{1-4x^2}}.$$

   □

2. Evaluate the following integrals:

   (a) $\int \frac{1}{1+x^2}dx$

   Solution. $\int \frac{1}{1+x^2}dx = \tan^{-1}(x) + C$ or $\arctan(x) + C$. □

   (b) $\int (x^{10} - \frac{1}{x^2} + e^x - \cos x)dx$

   Solution.

   $$\begin{aligned}\int (x^{10} - \frac{1}{x^2} + e^x - \cos x)dx &= \int (x^{10} - x^{-2} + e^x - \cos x)dx \\ &= \frac{x^{11}}{11} - \frac{x^{-1}}{-1} + e^x - \sin x + C \\ &= \frac{x^{11}}{11} + \frac{1}{x} + e^x - \sin x + C\end{aligned}$$

   □

(c) $\displaystyle\int \frac{1}{x} dx$

Solution. $\int \frac{1}{x} dx = \ln|x| + C$ □

(d) $\displaystyle\int \tan x dx$

Solution. Use the substitution $u = \cos x, du = -\sin x dx$:

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{u} du = -\ln|u| + C = -\ln|\cos x| + C \text{ or } \ln|\sec x| + C$$

□

(e) $\displaystyle\int x \sin x dx$

Solution. Use integration by parts formula: $\int u dv = uv - \int v du$. Set $u = x, dv = \sin x dx$. Then $du = dx, v = -\cos x$, and

$$\int x \sin x dx = -x \cos x - \int -\cos x dx = -x \cos x + \sin x + C$$

□

(f) $\displaystyle\int \frac{1}{x - x^2} dx$

Solution. Use partial fraction decomposition. The denominator factors as $x - x^2 = x(1 - x)$.

$$\begin{aligned} \frac{1}{x(1-x)} &= \frac{A}{x} + \frac{B}{1-x} \\ 1 &= A(1-x) + Bx \\ 1 &= A + x(-A + B) \end{aligned}$$

Set the x terms equal to each other, constant terms equal to each other to get the system of equations

$$\begin{cases} -A + B &= 0 \\ A &= 1 \end{cases}$$

The solution to this system of equations is $A = B = 1$. Therefore

$$\frac{1}{x(1-x)} = \frac{1}{x} + \frac{1}{1-x}.$$

Thus

$$\int \frac{1}{x(1-x)} dx = \int \left(\frac{1}{x} + \frac{1}{1-x} \right) dx = \ln|x| - \ln|1-x| + C.$$

Note: the solution can also be written as $\ln\left|\frac{x}{1-x}\right| + C$. □