Quiz 2 Solutions

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1. Show that the following differential equation is exact, and then solve the equation.

$$3e^{3x}y + (e^{3x} + 2y)y' = 0.$$

Solution. We are given $M = 3e^{3x}y$ and $N = e^{3x} + 2y$. Show that the equation is exact:

$$M_y = 3e^{3x}$$
$$N_x = 3e^{3x}$$

Since $M_y = N_x$, the equation is exact.

Integrate M with respect to x:

$$f(x,y) = \int M dx + g(y) = \int (3e^{3x}y)dx + g(y) = e^{3x}y + g(y)$$

Take the derivative of f with respect to y and set it equal to N:

$$\frac{\partial}{\partial y}(e^{3x}y + g(y)) = e^{3x} + 2y$$

$$e^{3x} + g'(y) = e^{3x} + 2y$$

Therefore $g'(y) = 2y$ and

$$g(y) = \int 2y dy = y^2.$$

Plug back into our formula for f:

$$f(x,y) = e^{3x}y + y^2$$

and set $f = C$:

$$e^{3x}y + y^2 = C.$$

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