Quiz 6 Solutions

Written by Victoria Kala
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1. Solve the following differential equation:

$$y''' - 2y'' + y' = 2\sin t$$

Solution. Find the homogeneous solution. The characteristic equation to $y''' - 2y'' + y' = 0$ is

$$m^3 - 2m^2 + m = 0 \quad \Rightarrow \quad m(m^2 - 2m + 1) = 0 \quad \Rightarrow \quad m(m-1)^2 = 0 \quad \Rightarrow \quad m = 0, 1 \text{ multiplicity } 2$$

Therefore $y_h = c_1 e^0 + c_2 e^t + c_3 t e^t = c_1 + c_2 e^t + c_3 t e^t$.
Use undetermined coefficients to find the particular solution. Since $g(t) = 2\sin t$, guess $y_p = A\sin t + B\cos t$. Calculate the derivatives:

$$\begin{aligned} y_p' &= A\cos t - B\sin t \\ y_p'' &= -A\sin t + B\cos t \\ y_p''' &= -A\cos t - B\sin t \end{aligned}$$

Plug into the differential equation:

$$-A\cos t - B\sin t - 2(-A\sin t + B\cos t) + A\cos t - B\sin t = 2\sin t$$

$$-2B\cos t + (2A - 2B)\sin t = 2\sin t$$

The coefficients on the left hand side must equal the coefficients on the right hand side. We get the system of equations

$$\begin{cases} -2B &= 0 \\ 2A - 2B &= 2 \end{cases}$$

which has the solution $A = 1, B = 0$. Thus $y_p = \sin t$.
The solution is therefore $y = y_h + y_p = c_1 + c_2 e^t + c_3 t e^t + \sin t$. □