Practice Midterm

Math 4B: Ordinary Differential Equations
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University of California, Santa Barbara

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Practice Midterm
10 Questions

Name: ____ Score : NA

Directions: Write your name in the space provided. There are no outside resources allowed − this includes books, notes, phones, and calculators. As determined by the instructor, an answer with no work shown may receive no credit. Unless otherwise specified, numbers included in a solution are not to be approximated, but instead expressed as exact numbers (i.e., in terms of square roots, multiples of π, etc.). All trigonometric expressions are to be evaluated.

Disclaimer: The content and level of difficulty of these practice questions are not guaranteed to be in correlation with the midterm nor final examination in any form.

1. Give an example of a third (3rd) order
   (a) nonlinear differential equation
   (b) linear differential equation
2. Verify $P = \dfrac{ce^t}{1 + ce^t}$ is a solution to the differential equation $\dfrac{dP}{dt} = P(1 - P)$.

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3. (a) Solve the following first order linear equation:

$$xy' + (1 + x)y = e^{-x} \sin 2x$$

(b) For the differential equation in part (a), determine the largest interval on which the existence and uniqueness theorem for first order linear differential equations guarantees the existence of a unique solution at the initial value $y(4) = 1$.

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4. Solve the following initial value problem:

$$\frac{dy}{dx} = x\sqrt{1-y^2}, \qquad y(0) = \frac{\pi}{6}$$

5. Solve the following separable equation:

$$\frac{dy}{dx} = \frac{xy + 2y - x - 2}{xy - 3y + x - 3}$$

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6. Solve the initial value problem

$$(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln(y))dy = 0, \qquad y(0) = e$$

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7. Solve the initial value problem

$$(1+x^2)\frac{dy}{dx} + 2xy = f(x), \qquad y(0) = 0$$

where $f(x) = \begin{cases} x, & 0 \le x < 1 \\ -x, & x \ge 1 \end{cases}$.

Hint: an integrating factor has already been multiplied through.

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8. Find the general solution of the following homogeneous higher-order differential equations:

(a) $y'' + 4y' + 7y = 0$ (use x as the independent variable)

(b) $y^{(3)} + 2y'' + y' = 0$ (use x as the independent variable)

(c) $2\dfrac{d^2u}{dt^2} - 5\dfrac{du}{dt} - 3u = 0$

(d) $\dfrac{d^4r}{ds^4} - r = 0$

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9. (a) Calculate $W(y_1, y_2, y_3)$ where $y_1 = e^x, y_2 = e^{2x}, y_3 = e^{3x}$.

(b) Suppose y_1, y_2 are solutions to the equation $x^2y'' + (2x - x^2)y' + y = 0$. Find $W(y_1, y_2)$.

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10. (a) The function $y_1 = x^2$ is a solution of $x^2y'' - 3xy' + 4y = 0$. Use the method of reduction of order to find a second solution y_2 to the differential equation on the interval $(0, \infty)$.

(b) Show that y_1 and y_2 form a fundamental set of solutions to the differential equation.

(c) Write the general solution of the differential equation using y_1 and y_2.

END OF EXAMINATION.