Practice Final

Math 4B: Ordinary Differential Equations
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Name: ____ Score : NA

Directions: Write your name in the space provided. There are no outside resources allowed − this includes books, notes, phones, and calculators. As determined by the instructor, an answer with no work shown may receive no credit. Unless otherwise specified, numbers included in a solution are not to be approximated, but instead expressed as exact numbers (i.e., in terms of square roots, multiples of π, etc.). All trigonometric expressions are to be evaluated.

Disclaimer: The content and level of difficulty of these practice questions are not guaranteed to be in correlation with the midterm nor final examination in any form.

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1. Solve the given initial-value problem:

$$\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}, \quad y(2) = 2$$

2. Solve the following differential equation:

$$(x+1)\frac{dy}{dx} = -(x+2)y + 2xe^{-x}$$

3. Consider the differential equation $(6xy^3 + \cos y)\frac{dy}{dx} + (2kx^2y^2 - x\sin y)dy = 0$.

   (a) Find the value of k so that the given differential equation is exact.

   (b) Solve the equation using your value of k from part (a).

4. Show that the equation

$$(x^2y + 4y)dy + xdx = 0$$

is not exact, then find the appropriate integrating factor to make the equation exact. Solve the initial value problem given $y(4) = 0$.

Hint: Put your equation in the correct form before you begin.

5. Show that $y_1 = x^2$ and $y_2 = x^2 \ln x$ are linearly independent solutions of the homogeneous equation $x^3y''' - 2xy' + 4y = 0$ on the interval $(0, \infty)$.

6. Use reduction of order to find a second solution y_2 to the differential equation

$$4t^2y'' + y = 0$$

given that $y_1 = t^{1/2}\ln t$ is a solution.

7. Solve the following initial-value problems:

(a) $y'' - 2y' + y' = 0, y(0) = 5, y'(0) = 10$

(b) $y''' + 12y'' + 36y' = 0, y(0) = 0, y'(0) = 1, y''(0) = -7$

(c) $y'' + 16y = 0, y(0) = 2, y'(0) = -2$

8. Suppose you are solving the equation $y'' + P(t)y' + Q(t)y = g(t)$, where $P(t)$ and $Q(t)$ are constants, using the method of undetermined coefficients. Complete the table below. Assume $g(t)$ has no function in common with the homogeneous solution y_h.

$g(t)$	Form of y_p
$3t^2 - 2$	(a)
(b)	Ae^{5t}
$6t^2 + 2 - 12e^{-2t}$	(c)
$e^{3x} \sin 4x$	(d)
(e)	$(At + B)e^{-3t}$
$1 - t^2 e^{-t}$	(f)
$3 \cos 2t$	(g)
(h)	$At + B$
$4t(1 + 3 \sin t)$	(i)

List your solutions for (a) - (i) below:

9. Solve the given differential equation using the method of undetermined coefficients.

$$y'' + 2y' = 3 + 4\sin 2t.$$

10. Solve the given initial-value problem.

$$y'' + 4y' + 4y = (3+t)e^{-2t}, \quad y(0) = 2, y'(0) = 5$$

11. Find the general solution of the given differential equation using variation of parameters.

$$y'' + y = \tan t$$

12. $y_1 = \cos(\ln t), y_2 = \sin(\ln t)$ are independent solutions of the equation

$$t^2 y'' + t y' + y = \sec(\ln t).$$

Find the general solution of the equation.

13. Solve the given initial-value problem:

$$\mathbf{x}' = \begin{pmatrix} \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

14. Solve the given initial-value problem:

$$\mathbf{x}' = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

15. Solve the given initial-value problem:

$$\mathbf{x}' = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$$

16. Solve $\mathbf{x}' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \mathbf{x}$.

17. Solve $\mathbf{x}' = \begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x}$.

18. Solve $\mathbf{x}' = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 6 & 0 \\ -4 & 0 & 4 \end{pmatrix} \mathbf{x}$.