Quiz 2 Solutions

January 31, 2016

1. Find the general solution of the ODE $ty' + 2y = t^2$.

   Solution. Write the equation in standard form: $y' + \frac{2}{t}y = t$. Here we see that $P(t) = \frac{2}{t}$, hence the integrating factor is

   $$\mu(t) = \exp\left(\int P(t)dt\right) = \exp\left(\int \frac{2}{t}dt\right) = e^{2\ln t} = e^{\ln t^2} = t^2.$$

   We then multiply our standard form equation by the integrating factor $\mu = t^2$:

   $$t^2\left(y' + \frac{2}{t}y = t\right) \quad \Rightarrow \quad t^2y' + 2ty = t^3.$$

   The left side can be written as a product rule: $\frac{d}{dt}(t^2y) = t^3$. We integrate both sides:

   $$\int \frac{d}{dt}(t^2y)dt = \int t^3dt \quad \Rightarrow \quad t^2y = \frac{t^4}{4} + C$$

   Then solve for y:

   $$y = \frac{\frac{t^4}{4} + C}{t^2} \quad \Rightarrow y = \frac{t^2}{4} + \frac{C}{t^2}.$$

   □

2. Find the general solution of the following ODE. Leave your answer as an implicit function of y.

   $$\frac{dy}{dx} = \frac{x}{x^2y^2 + 3y^2}.$$

   Solution. This is a separable equation. Factor the denominator:

   $$\frac{dy}{dx} = \frac{x}{y^2(x^2+3)}$$

   Separate the variables:

   $$y^2dy = \frac{x}{x^2+3}dx$$

   Integrate both sides:

   $$\int y^2dy = \int \frac{x}{x^2+3}dx \quad \Rightarrow \quad \frac{y^3}{3} = \ln|x^2+3| + C.$$

   This is the general solution in implicit form (i.e. we did not solve for y).
   (Note: use the substitution $u = x^2 + 3$ to evaluate the integral above.) □