# Math 6A Practice Problems I

Written by Victoria Kala
vtkala@math.ucsb.edu
SH 6432u Office Hours: R 12:30 − 1:30pm
Last updated 4/16/2016

1. For the following parametrizations:

   (i) Eliminate the parameter to find a Cartesian equation of the curve.

   (ii) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

   (a) $x = \sin t, y = \csc t,\ 0 < t < \pi/2$

   (b) $x = e^t - 1, y = e^{2t}$

2. Find an equation of the tangent line to the curve $x = 1 + \ln t, y = t^2 + 2$ at the point $(1, 3)$.

3. Find the exact length of the curve $x = e^t - t, y = 4e^{t/2}, -8 \le t \le 3$.

4. Find the derivative of the vector function $\mathbf{r}(t) = \sin^{-1} t\ \mathbf{i} + \sqrt{1 - t^2}\ \mathbf{j} + \mathbf{k}$.

5. Find the velocity, acceleration, and speed of a particle where the position function is given by $\mathbf{r}(t) = \sqrt{2}t\ \mathbf{i} + e^t\ \mathbf{j} + e^{-t}\ \mathbf{k}$.

6. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\ \mathbf{i} + e^t\ \mathbf{j} + te^t\ \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

7. Find the unit tangent vector $\mathbf{T}(t)$ to the curve $\mathbf{r}(t) = \langle te^{-t}, 2\arctan t, 2e^t\rangle$ when $t = 0$.

8. Consider the circular helix with vector equation $\mathbf{r}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t\ \mathbf{k}$.

   (a) Find the arc length of the helix $\mathbf{r}(t)$ from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

   (b) Reparametrize the helix $\mathbf{r}(t)$ with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing $t$.

   (c) Find the curvature $\kappa$ of $\mathbf{r}(t)$.

9. Find an equation of the line that

   (a) passes through the points $(1, 3, 2), (-4, 3, 0)$.

   (b) passes through the point $(1, 0, 6)$ and is perpendicular to the plane $x + 3y + z = 5$.

10. Determine whether the lines $L_1$ and $L_2$ are parallel, skew, or intersecting. If they intersect, find the point of intersection.

    (a) $L_1 : x = -6t, \quad , y = 1 + 9t, \quad z = -3t$
    $L_2 : x = 1 + 2s, \quad y = 4 - 3s, \quad z = s$

    (b) $L_1 : x = 1 + t, \quad y = -2 + 3t, \quad z = 4 - t$
    $L_2 : x = 2s, \quad y = 3 + s, \quad z = -3 + 4s$

11. Find an equation of the plane through the point $(6, 3, 2)$ and perpendicular to the vector $\langle -2, 1, 5 \rangle$.

12. Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$. (Hint: find the angle between the normal vectors of the planes.)

13. For the following functions:

    (i) Sketch the level curves $f(x, y) = k$ for the given $k$ values.

    (ii) Use your contour map from (i) to estimate $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ at the point $P(0, 0)$.

    (a) $f(x, y) = 6 - 3x - 2y$ for $k = -6, 0, 6, 12$

    (b) $f(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$

14. Find the limit, if it exists, or show that the limit does not exist:

    (a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{y^4}{x^4 + 3y^4}$

    (b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2 + y^2}$

15. Find all of the second partial derivatives of $f(x, y) = x^3y^5 + 2x^4y$. Does Clairaut's Theorem hold?

16. Find the linearization $L(x, y)$ of the function $f(x, y) = \dfrac{x}{x + y}$ at the point $(2, 1)$.

17. The length and width of a rectangle are measured as 30 cm and 24 cm respectively, with an error in measurement of at most 0.1 cm each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

18. Let $f(x, y, z) = xe^{2yz}$, $\mathbf{u} = \langle \frac{2}{3}, \frac{-2}{3}, \frac{1}{3} \rangle$.

    (a) Find the gradient of $f$.

    (b) Find the gradient at the point $P(3, 0, 2)$.

    (c) Find the rate of change of $f$ at $P$ in the direction of the vector $\mathbf{u}$. (In other words, find the directional derivative.)

19. Find the maximum rate of change of $f(x, y, z) = \dfrac{x + y}{z}$ at the point $(1, 1 - 1)$ and the direction in which it occurs.

20. (a) Find the local maximum and minimum values and saddles points of $f(x, y) = x^4 + y^4 - 4xy + 2$

    (b) Find the absolute maximum and minimum values of $f(x, y) = x^4 + y^4 - 4xy + 2$ on the set $D = \{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$.