Math 6A Practice Problems II

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1. Evaluate the line integral $\int_C xyzds$ where C is the curve parametrized by $x = 2\sin t, y = t, z = -2\cos t$, $0 \le t \le \pi$.

2. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given by the vector function $\mathbf{r}(t) = t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$, $0 \le t \le \pi$ and $\mathbf{F} = z\mathbf{i} + y\mathbf{j} - x\mathbf{k}$.

3. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.
   (a) $\mathbf{F}(x, y) = e^x \cos y\mathbf{i} + e^x \sin y\mathbf{j}$
   (b) $\mathbf{F}(x, y) = (\ln y + 2xy^3)\mathbf{i} + \left(3x^2y^2 + \dfrac{x}{y}\right)\mathbf{j}$

4. Let $\mathbf{F}(x, y, z) = y^2 \cos z\mathbf{i} + 2xy \cos z\mathbf{j} - xy^2 \sin z\mathbf{k}$, C be the curve parametrized by $\mathbf{r}(t) = t^2\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$, $0 \le t \le \pi$.
   (a) Show that $\mathbf{F}$ is conservative. (Hint: use curl)
   (b) Find a function f such that $\mathbf{F} = \nabla f$.
   (c) Use (b) to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

5. (a) Estimate the volume of the solid that lies below the surface $z = x + 2y^2$ and above the rectangle $R = [0, 2] \times [0, 4]$. Use a Riemann sum with $m = n = 2$ and choose the sample points to be the lower right corners.
   (b) Use the midpoint rule to estimate the volume in (a).
   (c) Calculate the exact volumes of the solid.

6. Evaluate the following integrals:
   (a) $\displaystyle\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dydx$
   (b) $\displaystyle\int_0^1 \int_0^1 \sqrt{s + t}\ dsdt$

7. Evaluate $\iint_D (x + y)\ dA$ where D is bounded by $y = \sqrt{x}, y = x^2$.

8. Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\ dxdy.$$

9. Evaluate $\iint_R (x+y)\, dA$ where R is the region that lies to the left of the y-axis between the circles $x^2+y^2=1, x^2+y^2=4$.

10. Verify Green's Theorem for $\int_C x^4dx + xydy$ where C is the triangular curve consisting of line segments from $(0,0)$ to $(1,0)$, $(1,0)$, to $(0,1)$, and $(0,1)$ to $(0,0)$ traversed in that order.

11. Evaluate $\int_C y^2dx+3xydy$, where C is the boundary of the semiannular region D in the upper half plane between the circles $x^2+y^2=1$ and $x^2+y^2=4$. (You may assume that C is positively oriented.)

12. Use a triple integral to find the volume of the solid bounded by the cylinder $y=x^2$ and the planes $z=0, z=4$, and $y=9$.

13. Evaluate $\iiint_E xydV$ where E is bounded by the parabolic cylinders $y=x^2$ and $x=y^2$, and the planes $z=0$ and $z=x+y$.

14. Evaluate $\iiint_E (x^3+xy^2)dV$ where E is the solid in the first octant that lies beneath the paraboloid $z=1-x^2-y^2$.

15. Evaluate $\iiint_E e^z dV$ where E is enclosed by the paraboloid $z=1+x^2+y^2$, the cylinder $x^2+y^2=5$, and the xy-plane.

16. Evaluate $\iiint_E xyzdV$ where E lies between the spheres $\rho=2$ and $\rho=4$ and the cone $\phi=\frac{\pi}{3}$.