Math 6B Practice Problems I

Written by Victoria Kala
vtkala@math.ucsb.edu
SH 6432u Office Hours: R 12:30 − 1:30pm
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1. Evaluate $\int_C y^2 dx + 3xydy$, where C is the boundary of the semiannular region D in the upper half plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (You may assume that C is positively oriented.)

2. Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = (\sqrt{x} + y^3, x^2 + \sqrt{y})$, and C consists of the arc of the curve $y = \sin x$ from $(0, 0)$ to $(\pi, 0)$ and the line segment from $(\pi, 0)$ to $(0, 0)$.

3. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.

4. Consider the vector field $\mathbf{F}(x, y, z) = yz\mathbf{i} + 2xz\mathbf{j} + e^{xy}\mathbf{k}$, where C is circle $x^2 + y^2 = 16, z = 5$ oriented counterclockwise when viewed from above.
   (a) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by finding an appropriate parametrization vector $\mathbf{r}(t)$.
   (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes' Theorem, and verify it is equal to your solution in part (a).

5. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$ where E is the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
   Note: to verify the theorem is true you need to show that $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} dV$; that is, you need to calculate both integrals and show they are equal.

6. Use the Divergence Theorem to calculate the surface integral $\iint \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S where $\mathbf{F}(x, y, z) = (\cos z + xy^2)\mathbf{i} + xe^{-z}\mathbf{j} + (\sin y + x^2z)\mathbf{k}$, and S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

7. Determine whether the sequence converges or diverges. If it converges, find the limit.
   (a) $a_n = e^{1/n}$
   (b) $a_n = n \sin\left(\frac{1}{n}\right)$
   (c) $a_n = 1 - (0.2)^n$
   (d) $a_n = n^2e^{-n}$
   (e) $a_n = \frac{(-1)^{n-1}n}{n^2 + 1}$
   (f) $a_n = \frac{\cos^2 n}{2^n}$
   (g) $a_n = \frac{n^n}{n!}$

8. Determine whether the series is convergent or divergent. State what test(s) you used to come to your conclusion.
   (a) $\sum_{n=1}^{\infty} \frac{1 + 3^n}{2^n}$
   (b) $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$
   (c) $\sum_{n=1}^{\infty} ne^{-n}$
   (d) $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$
   (e) $\sum_{n=1}^{\infty} \frac{1 + \sin n}{10^n}$
   (f) $\sum_{n=1}^{\infty} \frac{n + 1}{n\sqrt{n}}$
   (g) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{10^n}$
   (h) $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$
   (i) $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$
   (j) $\sum_{n=1}^{\infty} \left(\frac{n^2 + 1}{2n^2 + 1}\right)^n$

9. Use the Integral test to prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.