Math 33B Worksheet 6 (Linear System)

Name: ____ Score: __________

Circle the name of your TA: Ziheng Nicholas Victoria

Circle the day of your discussion: Tuesday Thursday

Overview: This week we focus on "linear system". Having a knowledge on linear algebra is highly recommended for learning this section. Our basic technique for solving linear ODE system is by finding the eigenvalues and eigenvectors of a matrix.

1. **Solving linear system of ODE**

   Consider 2×2 linear system $y' = Ay$, with

   (a) $A = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$ (c) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

   First try to find the eigenvalues and eigenvectors, write down $A = P\Lambda P^{-1}$. Then make substitution $y = Px$ to solve the linear system of ODE. Describe the long-term behaviour of your solutions. How does the eigenvalues influence the behaviour of solution?

2. **Mass-spring problem revisited**

Recall the equation of mass-spring system:

$$mx''(t) = -kx(t) - \mu x'(t) + F(t) \tag{1}$$

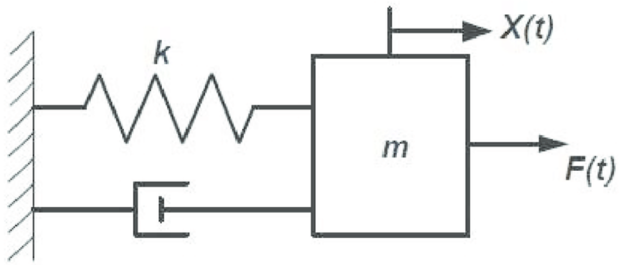

Figure 1: Mass on a spring

By subtitution $x'(t) = v(t)$, we get a 2D linear system of ODE:

$$\begin{cases} x'(t) = v(t) \\ v'(t) = -\dfrac{k}{m}x(t) - \dfrac{\mu}{m}v(t) + \dfrac{F(t)}{m} \end{cases} \tag{2}$$

We can rewrite it in matrix form

$$\begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\mu}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{F(t)}{m} \end{pmatrix} \tag{3}$$

(a) Suppose $F(t) = 0$, can you derive the same solution as we did in unforced harmonic motion? (Recall the 3 types of solutions with different damping coefficient.)

(b) (Optional) Suppose $\mu = 0, F(t) = F\cos(\omega t)$, can you derive the same solution as we did in forced harmonic motion? (Recall beats and resonance.)