Math 33B Worksheet 4 (Harmonic Motion)

Name: ______________________ Score: __________

Circle the name of your TA: Ziheng Nicholas Victoria

Circle the day of your discussion: Tuesday Thursday

Overview: Second order linear ODE with constant coefficients usually describes a harmonic motion. When there is no damping force, the solution is a periodic cosine funciton. When there is a damping force, the solution can be **underdamped**, **critically damped**, or **overdamped**, depending on the coefficients.

Now we are going to learn harmonic motion through two examples in physics.

1. **RLC Circuit**

Recall in first week's worksheet we discussed simple "RC circuit", where the circuit is composed of resistors (R) and capacitors (C). Now we introduce inductor (L):

	R	L	C
V=	IR	$L\frac{dI}{dt}$	$\frac{Q}{C}$

Figure 1 below illustrates the "RLC circuit". By Kirchhoff's law we have equation

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0 \tag{1}$$

(Here source voltage $V_{in} = 0$ because we only study homogeneous equation.)

Since $I = \frac{dQ}{dt}$, take derivative to equation (1), we get

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = 0 \tag{2}$$

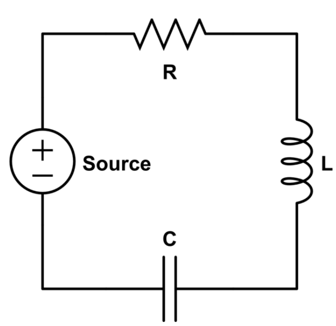

Figure 1: RLC circuit

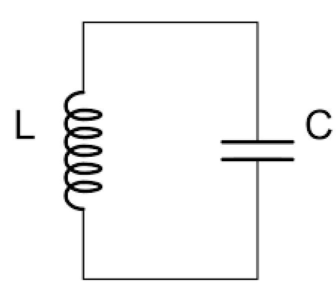

Figure 2: LC circuit

(a) If there is no resistor (R), it is "LC circuit" (as shown in figure 2), which satisfies

$$L\frac{d^2 I}{dt^2} + \frac{I}{C} = 0 \tag{3}$$

Given initial condition $I(0) = A\cos(\phi), I'(0) = -\frac{A}{\sqrt{LC}}\sin(\phi)$, try to find solution to (3). Explain the meaning of A, ϕ and $\omega_0 = \frac{1}{\sqrt{LC}}$.

(Hint: Try cosine solution.)

(b) Let's come back to RLC circuit. Try to find the general solution to equation (2). You may need to divide into cases according to parameters.

(Hint: Try complex exponential solution.)

(c) In part (b), classify solutions as "underdamped", "overdamped" and "critical damped". Try to explain their long term behavior. How does the behavior of solution depend on parameters?

2. **Mass on a Spring**

Let's look at another example of harmonic motion. Consider a spring-mass system (shown below). The motion can be described in following equation:

$$m\frac{d^2x}{dt^2} = -kx + D(x') + F(t) \tag{4}$$

where $D(x')$ is damping force, and $F(t)$ is externel force. Again, since we are talking about homogeneous equation, here assume $F(t) = 0$.

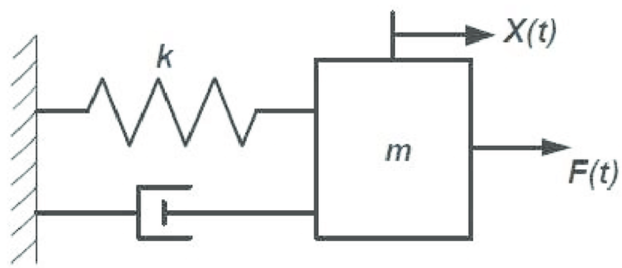

Figure 3: Mass on a spring

(a) If there is no damping force

$$m\frac{d^2x}{dt^2} = -kx \tag{5}$$

Given initial condition $x(0) = x_0, x'(0) = v_0$, try to find the solution to (5). What are the "amplitude", "phase angle" and "natural frequency"?

(b) Let's come back to equation (4) with damping force $D(v) = -\mu v$, i.e.

$$m\frac{d^2x}{dt^2} = -kx - \mu x' \tag{6}$$

Try to find the general solution to (6). You may need to divide into cases according to parameters.

(c) In part (b), classify solutions as ”underdamped”, ”overdamped” and ”critical damped”. Try to explain their long term behavior. How does the behavior of solution depend on parameters?

3. Compare your solution in problem 1 and 2. Did you see they are actually the same math problem? What general conclusion can you get about harmonic motion?