Math 33B Worksheet 9 (Review)

Name: ____ Score: __________

Circle the name of your TA: Ziheng Nicholas Victoria

Circle the day of your discussion: Tuesday Thursday

Directions: For Exercises 1-9, match its type using A-H. No term is used twice. Then solve Exercises 1-9.

A. First order separable equation

B. First order linear equation

C. Exact equation

D. Second order homogeneous equation

E. One dimensional phase portraits and stability

F. Second order nonhomogeneous equation, undetermined coefficients

G. Second order nonhomogeneous equation, variation of parameters

H. Two dimensional homogeneous linear systems and phase portraits

I. Higher order homogeneous linear systems

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1. ______ Solve $y'' - 2y' + y = e^t \arctan t$.

2. ______ Find the general solution of $(1+x)y' - xy = x + x^2$.

3. ______ Solve the following differential equations.

   (a) $2y'' - 5y' - 3y = 0$ (b) $y'' - 10y' + 25 = 0$ (c) $y'' + 4y' + 7y = 0$

4. ______ Find an explicit solution of the given initial value problem: $x^2 \frac{dy}{dx} = y - xy,\ y(-1) = -1$.

5. ______ For each of the following systems, (i) Draw a phase portrait for the system, (ii) classify the stability of (0,0), (iii) Find the general solution.

   (a) $\mathbf{X}' = \begin{pmatrix} 2 & 8 \\ -1 & -2 \end{pmatrix} \mathbf{X}$ (b) $\mathbf{X}' = \begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix} \mathbf{X}$ (c) $\dfrac{dx}{dt} = 2x + 3y$, $\dfrac{dy}{dt} = 2x + y$

6. ______ Solve $y'' - 2y' - 3y = 4x - 5 + 6e^{2x}$.

7. ______ Solve $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1-x^2)}, \quad y(0) = 2.$

8. ______ Consider the differential equation $\frac{dP}{dt} = P(a - bP)$ where a, b are positive constants.

   (a) Find the equilibrium points.

   (b) Draw a one-dimensional phase portrait, use this to classify the stability of the equilibrium points found in (a).

   (c) What is the expected behavior of $P(t)$ as $t \to \infty$?

9. ______ Solve $\mathbf{X}' = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{X},\ \mathbf{X}(0) = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}.$