Math 31AL Practice Problems I

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1. Consider the function

$$f(x) = \begin{cases} 1 - x, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x \geq 2 \end{cases}$$

   (a) Sketch the graph of $f(x)$.

   (b) Evaluate the following limits:

   i. $\lim_{x\to 0^-} f(x)$
   ii. $\lim_{x\to 0^+} f(x)$
   iii. $\lim_{x\to 0} f(x)$
   iv. $\lim_{x\to 1^-} f(x)$
   v. $\lim_{x\to 1^+} f(x)$
   vi. $\lim_{x\to 1} f(x)$
   vii. $\lim_{x\to 2^-} f(x)$
   viii. $\lim_{x\to 2^+} f(x)$
   ix. $\lim_{x\to 2} f(x)$

   (c) Is f continuous at $x = 0$, $x = 1$, or $x = 2$?

   (d) Is f a continuous function?

2. Find the constant k such that the following function is continuous:

$$f(x) = \begin{cases} kx^2 + 2x + 1, & 0 \leq x \leq 4 \\ 2 - kx, & x > 4 \end{cases}$$

3. Evaluate the following limits:

   (a) $\lim_{z\to 8} z^{2/3}$

   (b) $\lim_{x\to -3} x^{-2} + 2x$

   (c) $\lim_{u\to 4} \dfrac{\sqrt{u} - 1}{\sqrt{u+5} + 1}$

   (d) $\lim_{x\to -1} \dfrac{x^2 + 4x + 3}{2x^2 + x - 1}$

   (e) $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

   (f) $\lim_{x\to 1} \left(\dfrac{6}{x^2 - 1} - \dfrac{3}{x - 1} \right)$

   (g) $\lim_{t\to 0} \dfrac{(1 - \cos(4t))\sin(3t)}{t^2}$

   (h) $\lim_{x\to \frac{\pi}{2}} \dfrac{2x}{\sin x}$

   (i) $\lim_{x\to 0} \dfrac{\csc 2x}{\csc 5x}$

   (j) $\lim_{x\to \infty} \dfrac{3x^2 - 5x + 10}{21 - 6x^2 + x}$

   (k) $\lim_{x\to \infty} \dfrac{e^{2x} + 1}{e^{2x} - 1}$

   (l) $\lim_{x\to -\infty} \dfrac{8x + 7}{5 + x^2}$

4. Find the horizontal asymptotes for

$$f(x) = \frac{\sqrt{4x^2 + x + 1}}{3x - 1}$$

5. Use Squeeze Theorem to evaluate the limit

$$\lim_{x\to 0} (4^x - 1)\sin^2\left(\frac{1}{x}\right)$$

6. Use Intermediate Value Theorem to show that $x^2 - 10\sin x + 4 = 0$ has a solution on the interval $[0, \frac{\pi}{4}]$.

7. Let $f(x) = x^2 - x + 2$

   (a) Find $f(-1)$.

   (b) Find $f(-1+h)$.

   (c) Using (a) and (b), simplify $\dfrac{f(-1+h) - f(-1)}{h}$.

   (d) Now take the limit of what you found in (c), i.e. $\lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$.

   (e) What does the limit in (d) represent?

8. For $f(x) = x^{-1/2}$, use the formal definition of a derivative to show that $f'(x) = -\frac{1}{2}x^{-3/2}$.

9. Find the tangent line of $f(x) = x^2 + 5x - \sin x + 1$ at $x = 0$.

10. Find the derivatives of the following functions:

    (a) $f(x) = \cos x - x^{100}$ (b) $g(x) = x\sin x$ (c) $h(x) = \dfrac{x^2}{\cos x}$

11. Circle whether the statement is true or false:

    (a) T F In the square root $\sqrt{x^2+4}$, we can split up the square root, i.e. $\sqrt{x^2+4} = \sqrt{x^2} + \sqrt{4}$

    (b) T F In the square root $\sqrt{4x^2}$, we can split up the square root, i.e. $\sqrt{4x^2} = \sqrt{4}\sqrt{x^2}$

    (c) T F $(x+2)^2 = x^2 + 2^2$

    (d) T F $(x+2)^2 = x^2 + 4x + 4$

    (e) T F In the fraction $\dfrac{4}{x+4}$, we can cancel out the $4's$, i.e. $\dfrac{4}{x+4} = \dfrac{1}{x}$

    (f) T F In the fraction $\dfrac{4}{4(x+1)}$, we can cancel out the $4's$, i.e. $\dfrac{4}{4(x+1)} = \dfrac{1}{x+1}$

    (g) T F When adding or subtracting fractions, we need to make sure they have a common denominator.

    (h) T F When multiplying or dividing fractions, we need to make sure they have a common denominator.