Math 33B: Autonomous Equations, Second Order

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1. Let P be a population of animals. We speculate that the population develops as

$$P' = rP(C - P)$$

if there a finite amount of resources. Here, C is called the carrying capacity of the environment.

(a) Take $C = 100$. Draw a phase line of the system, identify the equilibria of the system, classify the equilibria as stable or unstable.

(b) Use your classification in (a) to draw a plot of various solutions P as a function of t. Include the equilibria solution in your plot, as well as solutions passing through $P = 50, P = 150$.

(c) Assuming we start with a positive population, what does our model predict the population will be after a long time? If we start off with 50 animals, is it possible to have 101? How do you know?

2. Consider the following differential equations:

A. $\dfrac{dy}{dx} + 5y = e^x$

B. $\dfrac{d^2y}{dx^2} + 5\left(\dfrac{dy}{dx}\right)^3 - 4y = e^x$

C. $y'' - 2y' + y = 0$

D. $(1-y)y' + 2y = e^x$

E. $\dfrac{d^2y}{dx^2} + \sin y = 0$

F. $\dfrac{d^3y}{dx^3} + x\dfrac{dy}{dx} - 5y = e^x$

G. $y'' + y^2 = 0$

H. $\dfrac{d^2y}{dt^2} + \ln(t)\dfrac{dy}{dt} - \arcsin(t)y = 0$

(a) Which of the above equations are 2nd order?

(b) Which of the above equations are 2nd order linear equations?

(c) Which of the above equations are 2nd order homogeneous linear equations?

3. y_1, y_2 are solutions of the given differential equation. Which of the following forms a linear independent set of solutions to the given differential equation?

(a) $y_1 = e^{3t}, y_2 = 5e^{3t},\ y'' - 9y = 0$

(b) $y_1 = \cos t, y_2 = \sin t,\ y'' + y = 0$

(c) $y_1 = e^{3t}, y_2 = e^{-3t},\ y'' - 9y = 0$

(d) $y_1 = e^t, y_2 = te^t, y'' - 2y' + y = 0$