Math 33B: First Order Equations

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Name: KEY

1. Let P be a population of animals. We speculate that the population develops as

$$P' = rP(C - P)$$

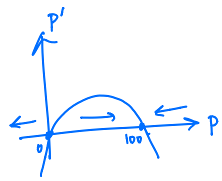
if there a finite amount of resources. Here, C is called the carrying capacity of the environment.

(a) Take $C = 100$. Draw a phase line of the system, identify the equilibria of the system, classify the equilibria as stable or unstable.

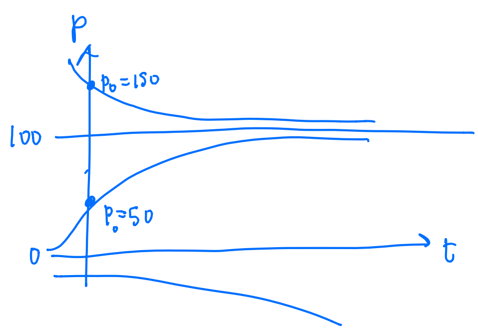
* I will take $r>0$ for this problem

$rP(100-P)=0$

$P=0$	$P=100$
unstable	stable



(b) Use your classification in (a) to draw a plot of various solutions P as a function of t. Include the equilibria solution in your plot, as well as solutions passing through $P = 50, P = 150$.



(c) Assuming we start with a positive population, what does our model predict the population will be after a long time? If we start off with 50 animals, is it possible to have 101? How do you know?

$P \to 100$ as $t \to \infty$

No, solution curves cannot cross each other

2. Consider the following differential equations:

A. $\frac{dy}{dx} + 5y = e^x$

B. $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x$

C. $y'' - 2y' + y = 0$

D. $(1-y)y' + 2y = e^x$

E. $\frac{d^2y}{dx^2} + \sin y = 0$

F. $\frac{d^3y}{dx^3} + x\frac{dy}{dx} - 5y = e^x$

G. $y'' + y^2 = 0$

H. $\frac{d^2y}{dt^2} + \ln(t)\frac{dy}{dt} - \arcsin(t)y = 0$

(a) Which of the above equations are 2nd order?

B, C, E, G, H

(b) Which of the above equations are 2nd order linear equations?

C, H

(c) Which of the above equations are 2nd order homogeneous linear equations?

C, H

3. y_1, y_2 are solutions of the given differential equation. Which of the following forms a linear independent set of solutions to the given differential equation?

(a) $y_1 = e^{3t}, y_2 = 5e^{3t}$, $y'' - 9y = 0$

$W = \begin{vmatrix} e^{3t} & 5e^{3t} \\ 3e^{3t} & 15e^{3t} \end{vmatrix} = 15e^{3t} - 15e^{3t} = 0$ not linear independent

(b) $y_1 = \cos t, y_2 = \sin t$, $y'' + y = 0$

$W = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t - (-\sin^2 t) = \cos^2 t + \sin^2 t = 1 \neq 0$ linearly independent

(c) $y_1 = e^{3t}, y_2 = e^{-3t}$, $y'' - 9y = 0$

$W = \begin{vmatrix} e^{3t} & e^{-3t} \\ 3e^{3t} & -3e^{-3t} \end{vmatrix} = -3e^0 - 3e^0 = -6 \neq 0$ linearly independent

(d) $y_1 = e^t, y_2 = te^t, y'' - 2y' + y = 0$

$W = \begin{vmatrix} e^t & te^t \\ e^t & e^t + te^t \end{vmatrix} = e^t(e^t + te^t) - e^t te^t = e^{2t} \neq 0$ linearly independent