1. Let $P$ be a population of animals. We speculate that the population develops as

$$P' = rP(C - P)$$

if there a finite amount of resources. Here, $C$ is called the carrying capacity of the environment.

(a) Take $C = 100$. Draw a phase line of the system, identify the equilibria of the system, classify the equilibria as stable or unstable.

![Phase line diagram](https://via.placeholder.com/150)

(b) Use your classification in (a) to draw a plot of various solutions $P$ as a function of $t$. Include the equilibria solution in your plot, as well as solutions passing through $P = 50, P = 150$.

![Plot diagram](https://via.placeholder.com/150)

(c) Assuming we start with a positive population, what does our model predict the population will be after a long time? If we start off with 50 animals, is it possible to have 101? How do you know?

$$\lim_{t \to \infty} P = 100$$

$N_o$, solution curves cannot cross each other.
2. Consider the following differential equations:

A. \( \frac{dy}{dx} + 5y = e^x \)

B. \( \frac{d^2y}{dx^2} + 5 \left( \frac{dy}{dx} \right)^3 - 4y = e^x \)

C. \( y'' - 2y' + y = 0 \)

D. \( (1 - y)y' + 2y = e^x \)

E. \( \frac{d^2y}{dx^2} + \sin y = 0 \)

F. \( \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 5y = e^x \)

G. \( y'' + y^2 = 0 \)

H. \( \frac{d^2y}{dt^2} + \ln(t) \frac{dy}{dt} - \arcsin(t)y = 0 \)

(a) Which of the above equations are 2nd order?

\( B, C, E, G, H \)

(b) Which of the above equations are 2nd order linear equations?

\( C, H \)

(c) Which of the above equations are 2nd order homogeneous linear equations?

\( C, H \)

3. \( y_1, y_2 \) are solutions of the given differential equation. Which of the following forms a linear independent set of solutions to the given differential equation?

(a) \( y_1 = e^{3t}, y_2 = 5e^{3t}, y'' - 9y = 0 \)

\[ W = \begin{vmatrix} e^{3t} & 5e^{3t} \\ 3e^{3t} & 15e^{3t} \end{vmatrix} = 15e^{3t} - 15e^{3t} = 0 \text{ not linear independent} \]

(b) \( y_1 = \cos t, y_2 = \sin t, y'' + y = 0 \)

\[ W = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t - \sin^2 t = \cos^2 t + \sin^2 t = 1 \text{ independent} \]

(c) \( y_1 = e^{3t}, y_2 = e^{-3t}, y'' - 9y = 0 \)

\[ W = \begin{vmatrix} e^{3t} & e^{-3t} \\ 3e^{3t} & -3e^{-3t} \end{vmatrix} = -3e^0 - 3e^0 = -6 \neq 0 \text{ independent} \]

(d) \( y_1 = e^t, y_2 = te^t, y'' - 2y' + y = 0 \)

\[ W = \begin{vmatrix} e^t & te^t \\ e^t & e^t + te^t \end{vmatrix} = e^t(e^t + te^t) - e^t(te^t) = e^t \neq 0 \text{ independent} \]