Math 31AL Practice Problems II

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Last updated November 21, 2019

1. Use chain rule to find the derivative of the following functions:
   
   (a) \( f(x) = \cos^2(x^3 + 1) \) 
   
   (b) \( g(x) = \sqrt{x + \sin x} \)

2. Find the equation of the tangent line at the given point using implicit differentiation:

   \[ 2x^{1/2} + 4y^{-1/2} = xy, \quad (1, 4) \]

3. Let \( f(x) = 2x^2 - 8x + 7 \).

   (a) Find the local extrema of \( f(x) \).
   
   (b) Find the absolute extrema of \( f(x) \) on the interval \([0, 5]\).
   
   (c) Find the absolute extrema of \( f(x) \) on the interval \([-4, 1]\).

4. Find all critical points of \( f \) and use the First or Second Derivative Test decide if they are local minima or maxima:

   \( f(x) = \frac{x^2}{x + 1} \)

5. Let \( f(x) = x^6 - 9x^4 \).

   (a) Find the intercepts of \( f \).
   
   (b) Find the critical points of \( f \) and intervals where \( f \) is increasing or decreasing.
   
   (c) Find the intervals where \( f \) is concave up or concave down. Does \( f \) have any inflection points?
   
   (d) Sketch the graph of \( f(x) \).

6. Let \( f(x) = \frac{x - 2}{x - 3} \).

   (a) Find the intercepts of \( f \).
   
   (b) Find the vertical asymptotes of \( f \).
   
   (c) Find the horizontal asymptotes of \( f \).
   
   (d) Find the critical points of \( f \) and intervals where \( f \) is increasing or decreasing.
   
   (e) Find the intervals where \( f \) is concave up or concave down. Does \( f \) have any inflection points?
   
   (f) Sketch the graph of \( f(x) \).

7. Victoria wishes to enclose a rectangular garden of area 1000m\(^2\). One side will be enclosed with a brick wall costing $90/m, the other three sides will be enclosed with a metal fence costing $30/m. What dimensions of the garden minimize the total cost?
8. Let \( f(x) = 9 - x^2 \).

(a) Sketch the graph of \( f(x) \) on the interval \([0, 3]\).

(b) Sketch the rectangles corresponding to \( L_3 \), then calculate \( L_3 \). Is this an underestimate or overestimate of the area under \( f(x) \)?

(c) Sketch the rectangles corresponding to \( R_3 \), then calculate \( R_3 \). Is this an underestimate or overestimate of the area under \( f(x) \)?

(d) Calculate a formula for \( R_N \).

(e) By taking the limit as \( N \to \infty \) of your answer in (d), calculate the area under \( f(x) \) on the interval \([0, 3]\).