Math 33B: Linear Systems

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Name: _____

1. Solve the linear system $\mathbf{x}' = A\mathbf{x}$ where

   (a) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$

2. Recall the equation of a mass-spring system:

$$mx''(t) + \mu x'(t) + kx(t) = F(t) \tag{1}$$

(a) For $F(t) = 0$, solve the system using second order methods. (*Note*: There are three types of solutions with different damping coefficients.)

(b) Using the substitution $x'(t) = v(t)$, we can rewrite (1) as a 2D linear system:

$$\begin{aligned} x'(t) &= v(t) \\ v'(t) &= -\frac{k}{m}x(t) - \frac{\mu}{m}v(t) + \frac{F(t)}{m}, \end{aligned}$$

which can then be written in matrix form

$$\begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\mu}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$

Find the solution to this system in the overdamped case , i.e. when $\mu^2 - 4km > 0$, when $F(t) = 0$. Is this the same solution as the overdamped case you found in (a)?