**Problem 1 Double integrals over rectangles.** Evaluate the following integrals:

(a) $\iint_R (x+y)dxdy$ where $R = \{(x,y) : 0 \leq x \leq 3, 0 \leq y \leq 1\}$

(b) $\int_0^1 \int_1^2 \frac{xe^x}{y} dydx$ (*Hint:* use integration by parts, $\int udv = uv - \int vdu$)

(c) $\int_0^1 \int_0^\pi r \sin^2\theta d\theta dr$ (*Hint:* use $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$)

**Problem 2 Double integrals over general regions.** Evaluate the following integrals:

(a) $\int_0^2 \int_x^{2x} xydydx$

(b) $\iint_D 2x\sqrt{y^2 - x^2}dxdy$ where $D = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq y\}$

**Problem 3 Switching the order of integration.**

(a) Sketch the region of integration and change the order of integration:

$$\int_0^4 \int_{x^2}^{16} f(x, y)dydx = \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x, y)dxdy$$

(b) Evaluate the integral by switching the order of integration:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dxdy$$

**Problem 4** **Average value.** The average value of a function $f(x, y)$ over a region $D$ is given by

$$f_{ave} = \frac{1}{A} \iint_D f(x, y) dA$$

where $A = \iint_D 1 dA$, i.e. the area of the region $D$. Use this formula to find the average value of the given function over the given domain.

(a) $f(x, y) = xy^2$, the rectangle with vertices $(-1, 0), (-1, 3), (2, 3), (2, 0)$

(b) $f(x, y) = x - y$, the triangle with vertices $(0, 0), (2, 0), (2, 2)$

**Problem 5 Triple Integrals.** Evaluate the following integrals:

(a) $\int_0^1 \int_0^1 \int_0^1 (9 - xz^2) dxdydz$

(b) $\int_0^2 \int_0^z \int_0^y 3ze^{-y^2} dxdydz$