Math 33A − Week 6

Written by Victoria Kala

February 11, 2020

Name: ____

1. Let $U = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \geq -2, y \leq 3 \right\}$. Show that U is not a subspace of $\mathbb{R}^2$.

2. Let $T(\mathbf{x}) = A\mathbf{x}$ where
$$A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 5 & -9 \\ 4 & 1 & 2 \end{pmatrix}.$$
   (a) Find bases for the image of T and the kernel of T. Describe them geometrically.
   (b) The dimension of $\text{im}(T)$ is 2. Does this mean $\text{im}(T) = \mathbb{R}^2$?

3. Let $T(\mathbf{x}) = A\mathbf{x}$. If A is a 3×5 matrix and its image of T is $\mathbb{R}^3$, does this mean that the kernel of T is $\mathbb{R}^2$?

4. Consider the following sets of vectors:
$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \right\}, \quad , T = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}, \quad U = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}$$
   (a) Which sets are linearly independent?
   (b) Which sets are a basis of $\mathbb{R}^3$?

5. Are the following sets of vectors linearly independent?
   (a) $\{1 + 2t, 4 + 5t + t^2, 1 - t + 7t^2\}$ in $\mathbb{P}_2$
   (b) $\{-1 + t, 2 + t^2, -3 + t + t^2, -1 - t - t^3, 2 + 3t^2 + t^3\}$ in $\mathbb{P}_3$
   (c) $\left\{ \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 4 & 0 \end{pmatrix} \right\}$ in $M_2, 2$

6. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be elements of a vector space V, and suppose the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent.
   (a) If the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, does this mean $\mathbf{w} \in \text{span}\{\mathbf{u}, \mathbf{v}\}$?
   (b) If $\mathbf{w} \in \text{span}\{\mathbf{u}, \mathbf{v}\}$, does this mean the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent?

7. Let $\mathbf{x} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.
   (a) $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ is a basis for $\mathbb{R}^2$. Find c_1, c_2 such that $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
   (b) The answer you find in (a) forms coordinate vector $[\mathbf{x}]_{\mathcal{B}} = [c_1, c_2]$. Fill in the blank using your answer in (a):
$$[\mathbf{x}]_{\mathcal{B}} = \underline{\hspace{3cm}}$$
   (c) $\mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\}$ also forms a basis for $\mathbb{R}^2$. What is $[\mathbf{x}]_{\mathcal{C}}$?