Problem 1 Let $f : \mathbb{R}^3 \to \mathbb{R}$ be an infinitely differentiable function. Show that the curl of ∇f is zero.

Problem 2 Let F be the vector field on $\mathbb{R}^2$ defined by

$$F(x, y) = \langle x^2, xy \rangle.$$

(a) Determine if F is conservative.
This can be done in many ways, here are some vauge hints:
- *Compute some path integrals*
- *Compute some partial derivatives*
- *Compute a curl*

(b) Compute

$$\int_{\mathcal{C}} F \cdot \mathrm{d}\vec{r}$$

where $\mathcal{C}$ is the unit circle around the origin, oriented counterclockwise.

(c) Compute div F.

Problem 3 Let F be the vector field on $\mathbb{R}^3$ defined by

$$F(x, y, z) = \langle -ye^{-x}, e^{-x} \rangle.$$

(a) Show that F is conservative.

(b) Find two different potential functions for F.

(c) Compute

$$\int_{\mathcal{S}} F \cdot \mathrm{d}\vec{r}$$

where $\mathcal{S}$ is the portion of the curve $x^4 + y^4 = 2$, oriented counter-clockwise, in the region $0 \le x \le y$.

Problem 4 One day, when Sandy is walking home with her bowling ball, she finds a beautiful hill whose shape is the graph of $f(x) = \frac{1}{1+x^2}$.

(a) Sandy decides to go to the top of the hill (at $(0, 1)$) and roll her bowling ball down. As she's climbing up, she passes her friend Ambrose, who says he'll catch the bowling ball when it rolls down. Ambrose is positioned at $(3, 0.1)$. Verify that Ambrose is on the hill.

(b) Sandy tosses the ball down the hill and it follows some trajectory $r(t)$, $0 \le t \le 1$ such that $r(0) = (0, 1)$ and $r(1) = (3, 0.1)$. Assuming that the force of gravity on the ball as it rolls down the hill is always $\langle 0, -1 \rangle$, find the work done by gravity on the ball during its trip down the hillside.

(c) Explain how and why the specific shape of the hill and the specific trajectory r down the hill doesn't affect the answer in the previous part.

Problem 5 An electron is placed in a newfangled particle decelerator. The electron starts at $(1, 0)$ with velocity $(0, 1)$, and experiences a force of $F = \langle -x, -y \rangle$ as it moves.

(a) Sketch the path the particle will take. This doesn't need to be precise, just give a rough idea.

(b) Prove that F is conservative by showing its curl is 0.

(c) Find a potential function for F.

(d) Determine the total amount of work done on the particle by F along its path. *Hint: it's not neccesary to parametrize the electron's path!*