Problem 1 Use geometric reasoning to find $\int_S \mathbf{F} \cdot \mathbf{dS}$ where $\mathbf{F}$ and $\mathbf{S}$ are the following:

(a) $\mathbf{F} = \langle 1, 0, 0 \rangle$ and $\mathbf{S}$ is the union of two squares $\mathbf{S}_1$ and $\mathbf{S}_2$ given by: $\mathbf{S}_1 : x = 0, 0 \leq y \leq 1, 0 \leq z \leq 1$ and $\mathbf{S}_2 : z = 0, 0 \leq x \leq 1, 0 \leq y \leq 1$ where $\mathbf{S}_1$ is oriented in the positive x direction and $\mathbf{S}_2$ in the positive y direction.

(b) $\mathbf{F} = \langle 1, 1, 0 \rangle$ and $\mathbf{S}$ is the square given by $\mathbf{S} : x = 0, 0 \leq y \leq 1, 0 \leq z \leq 1$.

(c) $\mathbf{F} = \langle 1, 0, 0 \rangle$ and $\mathbf{S}$ is the cylinder given by $x^2 + y^2 = 1$ from $z = 0$ to $z = 1$ oriented outwards.

(d) $\mathbf{F} = \langle x, y, 0 \rangle$ and $\mathbf{S}$ is the cylinder given by $x^2 + y^2 = 4$ and $1 \leq z \leq 3$ oriented outwards.

Problem 2 Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle xyz, xyz, xyz \rangle$ and S is the five faces of the cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ missing $z = 0$ that is oriented outwards. *Hint: It is enough to just calculate one of the faces and multiply the result by 3. Why?*

Problem 3 Use green's theorem to calculate $\int_C x^2 y dx + (y - 3) dy$ where C is the perimeter of the rectangle with vertices $(1, 1), (4, 1), (4, 5)$ and $(1, 5)$ oriented counter-clockwise.

Problem 4 Compute $\int_{\partial D} \left(\sin x - \frac{y^3}{3} \right) dx + \left(\sin y + \frac{x^3}{3} \right) dy$ where D is the annulus given in polar coordinates by $0 \leq \theta \leq 2\pi, 1 \leq r \leq 2$.

Problem 5 Consider the vector field $\mathbf{F} = \langle y, 2x \rangle$. Suppose we have two paths γ_1 and γ_2 that both start and end at the same point. How do the two line integrals of $\mathbf{F}$ differ along the two paths?