Math 151A - Spring 2020 - Week 2

Office hours are Mondays 5-6pm PT and Thursdays 2-3pm PT.

From last week: `format long` and `format short` in Matlab do not affect precision.

Today: Bisection Method

Example. (Similar to Exercises 3 and 4 on homework)
Consider the function $f(x) = x^3 + 4x^2 - 10$ on the interval [1,2].

(a) Show that $f(x)$ has exactly one root on $[1, 2]$ without solving the equation.

Proof. f is continuous. $f(1) = -5 < 0$, $f(2) = 14 > 0$.
By Intermediate Value Theorem there is a $p \in (1,2)$ such that $f(p) = 0$.

$f'(x) = 3x^2 + 8x > 0 \quad \forall\, x \in [1,2]$

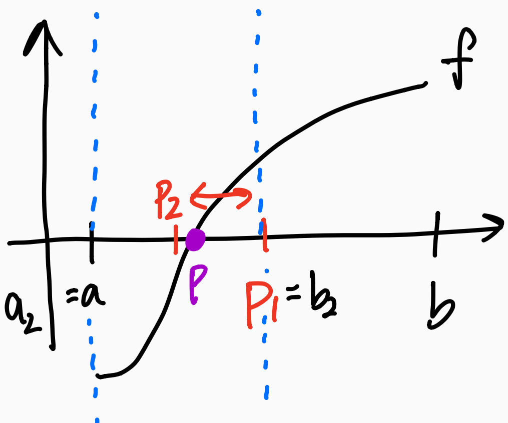
f is strictly increasing in $[1,2]$ $\Rightarrow$ exactly one root in $[1,2]$.

10^{-5} tol = 1e-5

Example. (Similar to Exercises 3 and 4 on homework)
Consider the function $f(x) = x^3 + 4x^2 - 10$ on the interval $[\underset{a}{1},\underset{b}{2}]$.

(b) Consider the bisection method algorithm starting on the interval $[1,2]$. Find the minimum number of iterations required to approximate the solution with an absolute error of less than 10^{-3}.

1e-3 = 1×10^{-3}



Observations

$$b_2 - a_2 = \frac{1}{2}(b_1 - a_1)$$
$$\vdots$$

- $b_n - a_n = \frac{1}{2}(b_{n-1} - a_{n-1}) = \dots = \frac{1}{2^{n-1}}(b-a)$

$$|p_2 - p_1| = \frac{1}{2}|b_2 - a_2|$$
$$\boxed{|p_n - p_{n-1}| = \frac{1}{2}|b_n - a_n|}$$

$$|p - p_1| \le \frac{1}{2}|b-a|, \quad |p - p_2| \le \frac{1}{2}|b_2 - a_2|, \dots$$

- $|p - p_n| \le \frac{1}{2}|b_n - a_n| = \frac{1}{2^n}|b-a| = \frac{1}{2^n} < 10^{-3}$

$$2^n > 10^3$$
$$n > \frac{\log 10^3}{\log 2} \approx 9.96 \text{ ish}$$

$$\boxed{n = 10 \text{ iterations}}$$

Example. (Similar to Exercises 3 and 4 on homework)
Consider the function $f(x) = x^3 + 4x^2 - 10$ on the interval [1,2].

(c) Now program a bisection algorithm to verify this. In particular, create three figures.

- In the first figure, plot the values $|p - p_n|$ on the y-axis, and the iteration number in the x-axis.
- In the second figure, plot $|p - p_{n-1}|$ in the y-axis and the iteration number in the x-axis.
- In the third figure, plot the values for $|f(p_n)|$ on the y-axis and the iteration number in the x-axis.

Do your experiments coincide with (b)?