Office hours are Mondays 5-6pm PT and Thursdays 2-3pm PT.

From last week: format long and format short in Matlab do not affect precision.

Today: Bisection Method
**Example.** (Similar to Exercises 3 and 4 on homework)
Consider the function \( f(x) = x^3 + 4x^2 - 10 \) on the interval \([1, 2]\).

(a) Show that \( f(x) \) has exactly one root on \([1, 2]\) without solving the equation.

*Proof.* \( f \) is continuous. \( f(1) = -5 < 0, \ f(2) = 14 > 0 \).

By Intermediate Value Theorem there is a \( p \in (1, 2) \)

such that \( f(p) = 0 \).

\( f'(x) = 3x^2 + 8x > 0 \ \forall \ x \in [1, 2] \)

\( f \) is strictly increasing in \([1, 2]\) \( \Rightarrow \) exactly one root

in \([1, 2]\).
Example. (Similar to Exercises 3 and 4 on homework)
Consider the function \( f(x) = x^3 + 4x^2 - 10 \) on the interval \([1,2]\).

(b) Consider the bisection method algorithm starting on the interval \([1,2]\). Find the minimum number of iterations required to approximate the solution with an absolute error of less than \(10^{-3}\).

\[
\frac{b_2 - a_2}{2} = \frac{1}{2} (b_1 - a_1) \\
\frac{b_n - a_n}{2} = \frac{1}{2} (b_{n-1} - a_{n-1}) = \ldots = \frac{1}{2^{n-1}} (b - a)
\]

\[
|p - p_1| \leq \frac{1}{2} |b - a| \\
|p - p_2| \leq \frac{1}{2} \left| \frac{1}{2} (b_2 - a_2) \right| \\
\ldots
\]

\[
|p - p_n| \leq \frac{1}{2} \left| \frac{1}{2} \left( \frac{1}{2} \left( \ldots \left( \frac{1}{2} (b_n - a_n) \right) \ldots \right) \right) \right| = \frac{1}{2^n} |b - a| = \frac{1}{2^n} < 10^{-3}
\]

\[n > \log_{10} \frac{3}{10^{-3}} \approx 9.99_{\text{ish}}\]

\[n = 10 \text{ iterations}\]
Example. (Similar to Exercises 3 and 4 on homework)
Consider the function $f(x) = x^3 + 4x^2 - 10$ on the interval $[1,2]$.

(c) Now program a bisection algorithm to verify this. In particular, create three figures.

- In the first figure, plot the values $|p - p_n|$ on the $y$-axis, and the iteration number in the $x$-axis.
- In the second figure, plot $|p - p_{n-1}|$ in the $y$-axis and the iteration number in the $x$-axis.
- In the third figure, plot the values for $|f(p_n)|$ on the $y$-axis and the iteration number in the $x$-axis.

Do your experiments coincide with (b)?